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ABSTRACT

This report resulted from work with primary grade children and teachers in Kankakee, Illinois. Essays by four resource persons and two observers are included, each expressing insights and feelings in order to share ideas with resource persons for primary mathematics teaching in other schools. A main objective is to represent the ways elementary school teachers think while teaching mathematics. The essays consider: general comments on learning and working with teachers, some difficulties experienced in teaching mathematics, classroom etiquette or management, a lesson on multiplication with fractions, lessons with Cuisenaire rods, equity in mathematics education, an overview of instructional policy, an in-service teacher training project, an idea about the mode of learning school requires, a report about a lesson on numeration, developing schema when teaching place value, and suggestions for resource persons. A list of references is included. (MNS)

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PEDAGOGICAL DIALOGS IN PRIMARY SCHOOL MATHEMATICS

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August, 1980

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photo--Ivan Ruiz

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PREFACE

During the past three years, I have been working with colleagues and graduate students at the Stueben Primary Center in Kankakee, Illinois. For two years, I coordinated clinical interviews of primary grade children on their conceptions of number and numeral, under an NIE contract, and in the past semester we have been working as resource persons with six volunteer teachers on problems of mathematics teaching. We have learned a great deal about why arithmetic is so very difficult for primary grade children (as compared with other subjects) and why the recommendations of specialists for improvement of arithmetic teaching are so little trusted by primary school teachers. This knowledge is primarily in the form of personnal knowledge of the four resource persons and two observers, and we report it here in the form of a baker's dozen essays by these six people. At this stage, we are more interested in having each writer express the insights and feelings they have been developing in Steuben Primary Center than in attaining consistency of theory and format. What we have put together between these covers is a written form of pedagogical dialog among ourselves, not unlike the dialogs we have been having with teachers.

The teachers' viewpoints are mainly represented here by the interview report prepared by one of our observers, Carol Chapelle. We regret there was no time to ask teachers to prepare written responses to our various essays. The real response from the teachers will come as they find ways or fail to find ways of using the ideas we have tried to put down here. However, the primary purpose of this report is not to evaluate our work, but to share it with those who are or would be resource persons for primary mathematics teaching in other schools. It is their response that counts most in terms of how useful our ideas are. However, it is the six teachers who, in a fundamental sense are the authors of this report, for they have been our teachers. If we have not

learned what they wished to teach us or have not solved the problems they wanted us to work on, we hope to have the chance to try again next year.

We chose to work in a Kankakee school because, with more than fifty percent of minority children enrolled, the teachers are facing some of the problems that plague inner-city schools in large, metropolitan areas. Also, the administration of the Kankakee Schools is making a determined effort to upgrade the quality of the program, to retain a significant number of white middle-class children and to establish a form of accountability for the effective education of all children. We are deeply grateful to the children, teachers, and administrators of the Steuben Primary Center for their cooperative spirit and willingness to help us learn what we, as resource persons, need to know about their work. We are especially indebted to the Superintendent of the Lincoln Consolidated Schools, near Ypsilanti, Michigan, for releasing Harold Taylor from his duties to work at Steuben Primary Center once a month as well as for a similar arrangement at Thomas Paine School in Urbana.

We are also very much indebted to the College of Education of the University of Illinois at Urbana/Champaign for many kinds of support for our work. Besides the helpful criticism of colleagues and students, the Committee on Culture and Cognition has provided work space, secretarial assistance, and the use of its video-tape analysis laboratory, and the Bureau of Educational Research has provided very faithful and expert clerical support, and especially through the Sciences Tutoring Laboratory, financial support for travel and research assistants. Finally, the Ford Foundation came to our aid with funds to permit employing a consultant, a technical assistant, and two observers. Without this assistance, the present report would not have been possible.

--Jack Easley

The Authors

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Harold Taylor is a Title I Remedial Math Teacher for the Lincoln Elementary School in Ypsilanti, Michigan, where he serves as director of the mathematics laboratory and as a mathematics resource person to the approximately sixty teachers in the school. He was formerly affiliated with the Booker T. Washington Laboratory School in Champaign, Illinois, and has pursued doctoral studies in mathematics education at the University of Illinois.

Introduction

One object of our inquiry is to represent the ways elementary school teachers think while teaching mathematics. Just as Piaget gave respectability to the pre-operational thinking of children (e.g., non-conservation) that tends to disturb many teachers and parents, by giving it the high status of functional and regulative structures from which later operations of the concrete-operational thinking are derived, so we would like to represent, as clearly as possible, the thinking of teachers who follow the more common teaching practices, and the developmental relationships they may or may not have to theoretical pedagogics of mathematics specialists.

We see "traditional" mathematics teaching in the elementary school as a form of thought which has a potential for development in directions that are very promising. However, most mention of such traditional forms of teaching in the literature on research are critical in the extreme and verge on sneering. This attitude on the part of the researchers and other specialists seems to arise partly from the enthusiasm of discovering what seem to the specialist to be much better ways of teaching, partly from ignorance of the practical constraints that block achievement of the ways they recommend, and partly from failure to recognize the positive values associated with the traditional forms that would apparently be sacrificed in changing to the recommended forms. Furthermore, there is the question of how those cognitive systems of teachers which have to operate spontaneously "in real time" under the stress of their classrooms, change or develop. From current theory, there is little reason to suppose these systems are related to the kinds of changes that can occur in the thinking of teachers enrolled in education classes, for such systems only have to operate on

specific demand, with delays permitted to gather together or construct a response to a professor's question. It has been argued that theoretical knowledge of teaching could operate in teachers the way it does in doctors or engineers who learned it in classrooms. However, with respect to primary school teachers, at least, it is clear to us that most situations have to be responded to immediately, without time to reflect and construct a response from theoretical principles.

For example, to change from thinking of arithmetic as based on counting to thinking of it as based on domino patterns, would be like trying to have secretaries convert from the standard typewriter keyboard to the Dvorak keyboard (Neill, 1980) while carrying on their duties in the office. It is only reasonable to expect that the office work would be slowed down so substantially that people dependent on quick typing would become very irritated. Being adults, they may not go out of control, but no one would propose a busy office as an ideal place for such a conversion to take place. Instead, we would allow typists a few months practice with the Dvorak system away from the office, to internalize the new system thoroughly before using it in the office. If a teacher, inspired by a workshop and new instructional materials, attempted to make such a conversion in his or her habitual responses to the mathematical ideas of primary children, chaos would result.

The resource persons in this project have experienced the phenomenon which Harold Taylor identified as like attempting to hold 25 corks under water with two hands. This was not only experienced by those of us who were relatively inexperienced in primary school teaching, but on one occasion by Hal Taylor, too. Every teacher develops techniques for keeping a modicum of order. Because the children learn what these techniques are, or at least

can accommodate to them, what works for a resource person in one classroom may not work at all in another. While we wanted to show teachers that some different techniques would work with their own pupils, and they wanted to see something different that worked, we were not always instantly successful. In fact, only Hal Taylor was successful most of the time when he first worked with a new class -- presumably because he has spent so much time teaching demonstration classes in so many different classes in his own school during the past five years. We wanted to see teachers trying out a new way of teaching mathematics to see whether the class would fall apart. So it was important that the same thing often happened to us, and they could observe how we coped with the problem the next time, if indeed we did.

The way classroom systems of self-control work is often very subtle. In only one or two of the teachers' classes did we have the impression that we saw external control systems operating clearly, and that mostly when it was not working well. For example, on one occasion, a teacher who did not have the attention of more than a handful of the class, simply went on teaching, calling on students individually, ignoring the disturbance of those talking to each other and getting out of their seats. After twenty minutes or so, the entire class was involved in the math lesson conducted by the teacher from the front of the room. This was surprising! Self-control took over when external control failed.

More often, teachers waited until they had silence and everyone facing front before they began a math lesson. On another occasion, a teacher spoke very sharply to those who were not quiet and attentive, and physically pulled one or two offending children out of their seats and set them down elsewhere in the room. These, however, are the exceptions. Mostly, there is a quiet, orderly pattern, in which the teacher appears to spot potential disruptions

evidenced by a sideways glance, a stretch, or a mumbling under the breath, and then call on that student for an answer to the next question.

Early in the school year most of the teachers seemed to have required each pupil to display their first name in large print on the part of the desk facing front, but some of these had been torn off by the time we began to work in the Spring semester. Clearly, it was important for the teacher to be able to call out a child's name instantly as needed. We found these name tags essential when we taught the classes, because we did not have the teacher's command of names, and names are much better than pointing, although even that can be excused when the teacher is brand new in the class. However, there is a subtle conflict for the teacher between thinking of the subject matter and thinking of what might be going wrong in classroom decorum. While resource persons tended to be more flexible and creative in mathematical thinking, the classroom teacher had a definite advantage over us in sensing something amiss in pupil behavior. When resource persons taught classes, it was clear that the teacher was often still keeping an eye on the class behavior, as with a whispered name or a silent pat on the shoulder, she responded to someone's violation of classroom decorum that wasn't being handled by the resource person up front.

What emerges from the struggle of the resource persons to show teachers something workable, but different from their own system, is the problem of controlling the excitement of mathematical discovery. Both the children and the resource person teaching become excited when, for example, children began to work on a mystery presented to them by Hal Taylor. Why does $\begin{array}{r} 15 \\ +15 \\ \hline 30 \end{array}$? There are only two tens shown in the tens column and the answer shows three tens. The hands go up, "I know, I know," and yet many children are mystified.

"Give everybody a chance to find out where that extra ten came from," said the resource person. Here, a great deal of self-control is required on the part of the children, both those who had discovered the missing ten and those who had not. The resource person really had to stay on top of the class, yet the excitement is exactly what he was trying to create and success was here if only it didn't get out of hand and spoil the whole thing. Brandau and Easley (1980) point out both that studies of problem solving often describe the profound and unpredictable emotions that result from having a sudden idea in response to a serious problem-solving effort, and the trouble that these emotions can cause for classroom teachers involved in creative problem solving.

Perhaps it is learning how to prevent or contain the excesses of emotion stemming from mathematical problem solving that teachers find the biggest obstacle to the adoption of Hal Taylor's group methods for creative problem solving--once they have been demonstrated and their motivating effect on children has been seen. I was so suprised to see a class that had evidently been used to taking turns in thinking through a problem lose control when they were invited to all think through the problem at once. That is, they clammored to each be allowed to say the answer, and of course once some one person said it, there was no joy in anyone else saying "me too." This contrasted sharply with another class in which, whenever one person got a difficult problem right, regardless of what others may have gotten -- or perhaps especially if they had the same answer -- they cheered. Here was a more self-disciplined class as contrasted with a class used to being primarily controlled by the teacher. Will the second teacher pick up Taylor's method easier than the first?

We did not go into the classrooms to expose teachers' methods and problems. We went there to help them solve problems. In the process of our becoming

involved, their problems became our problems too. We often did not have ready-made solutions to show them. Even if we showed them something that went well, it was not at all clear what problems it was a solution to. Perhaps, at best, it was a stimulus to rethink the problems we both knew. Our pedagogical dialogs were therefore not strictly analogous to the Piagetian clinical interview, in which the subject is thinking out loud about the objects being manipulated in front of the interviewer. Sometimes the teacher thought out loud about some bit of pedagogy in which we had both participated. However, we thought out loud too, for we were struggling to understand the bit of pedagogy too, and we imparted our own ideas and re-examined them. This is why we call our work pedagogical dialogs, rather than participant observation, which it is, or exploratory clinical interviews which it comes close to.

It's so easy to assume that teachers' schedules are inflexible, and it's so easy to forget the feelings of ownership of children for place and time. For example, we were having a bit of difficulty in getting our key demonstration teacher, Hal Taylor, scheduled into every classroom where his services were wanted because of overlaps in the schedule of math classes. So, we once proposed a change in the schedule for math classes that involved three teachers. They all agreed to the change, although more or less reluctantly. For example, one said, "As long as it's just for today." We accepted their agreement, but were probably thinking that teachers were more rigid or possessive of their time slots than necessary. When the revised schedule was put in operation, however, we found out why teachers might instinctively value a fixed schedule. The children in one class objected vehemently to having their schedule changed. They were suspicious that they were going to be cheated, if not in time, at least of their rightful time slots for this and that class. The unhappy pupil reaction was not something wanted, so, we hit upon a better plan; we divided

up the six teachers among three resource persons, even when Hal Taylor was there as a fourth person. The expertise of Taylor could not easily be shared directly with six teachers. What was more important was that each teacher had an on-going relation with a resource person, an occasional casual relationships with others, and then Hal Taylor's expertise could be shared with the teachers, in different ways, indirectly or directly.

On another occasion, I ignored children's possessive attitudes toward their space, namely their desks. The class was divided up into four groups to do a measurement activity. Four or five children were in each group. Each group needed a work space formed by pushing together four of their desks. I took the four nearest desks for my group of five, not bothering to find out even whose desks they were. I simply pushed them together, despite two students who wanted to know who was going to be working at their desk. From time to time, during the work, children came over to get something out of their desk, and at the end one rather brusk boy threatened to get even with another boy who had been sitting at his desk. Shades of The Three Bears, I thought and proceeded to instruct the threatening boy that it was my decision that his desk had been used by another, and that I would hold him responsible if he did anything to the boy who had sat there. However, I learned a deep lesson from these episodes about the invasion of property and time which children come to feel, in some sense, belongs to them. There are many constraints we ignore.

I am thinking over the many things I saw teachers do in their classes. There were quizzical looks, enthusiastic eagerness, worried frowns, and many, many, others. What stands out in my mind as deserving the most sympathetic attention of resource persons is what I would call "persuasive hopeful questioning." A teacher (and sometimes a resource person too) has an eye on a particular goal. For example, that a particular mathematical operation will be performed and

called by the name it is known by. Since this knowledge is clear in the teacher's mind, but the pupils' struggles to get to it are not, there comes a point at which a teacher has to resort to persuasive, hopeful questioning.

Here is an example: "Now adding thirties is just like adding threes, isn't it?" (silence) "Why is adding thirties just like adding threes, June?" (silence) "Because there are three what in thirty, William?" (ten) "Tens, yes. There are three tens in thirty, so adding thirties is just another case of adding three things, three apples, three children, or whatever. Do you see that?" (silence)

Now, the point I wish to make about persuasive, hopeful questioning is not that it should be avoided. We cannot avoid it when we resource persons teach. It is that it was surprising for me to see it and to realize that it may have a function despite some theoretical objection against it. This means that the theories that object to it are at least, over-simplified. If you see it, when there is no evidence that any child in the class understands the teacher's idea, you wonder about indoctrination and the building up of a conventionalist view of mathematics. However, if you are teaching, and you have managed to get a handful of children on the verge of seeing the point, and the bell for recess is about to ring, you are impelled to try some persuasive hopeful questioning, so as not to let the children go away feeling that they never got the point of the lesson. Letting the math class end in a state of confusion is obviously very bad for the children's morale, teacher morale, and could lead to complaints funneled back to the principal from parents. What alternative do you have? An honest statement that "Well, we didn't get to the main idea but we'll make it, tomorrow," is not satisfactory. Somehow the lesson has to be induced, like a pot being thrown on a wheel or, better, a meal being prepared, to all get together at the right time. Only

the pot can be thrown back in the clay bin as a failure with no disgrace attached.

What follows are the representations of the resource persons who worked in Steuben School in the Spring of 1980 of what they learned and how they now see the problem of helping teachers of primary school mathematics make improvements.

-- Jack Easley

Reference

Neill, Shirley Boes (1980) "Dvorak vs. Qwerty: Will Tradition Win Again?" Phi Delta Kappan, 61:671-673.

An Inspection of Some Difficulties Experienced in Teaching Mathematics

Aimee Grieb

The immediate problem - teaching children subtraction

The first topic we were asked to help with was subtraction involving borrowing or renaming. The teacher I was working with was specifically complaining that her students were not doing it correctly. A very common error when they were doing subtraction is illustrated in the following problem:

$$\begin{array}{r} 100 \\ -99 \\ \hline 199 \end{array}$$

Essentially what was occurring was that the smaller digit was being subtracted from the larger one irregardless of which is above or below, i.e., no matter if it is from the quantity to be subtracted from or the quantity to be subtracted. Notice too that the extreme is larger than the initial amount of 100. Logical inspection would indicate that something is wrong. Either the students are not considering what this is saying (that a quantity can become larger when a quantity is subtracted from it), or, they recognize the inconsistency and simply do not bother to resolve it.

Other types of errors on subtraction were also occurring, but it was not due to the teacher's inaccurate teaching of subtraction. She, and other teachers, have a very effective method for doing subtraction, and one which most adults use automatically. Using this problem $\begin{array}{r} 435 \\ -198 \\ \hline \end{array}$ as an example, I will outline the method being taught:

A. Of primary importance is to do the subtraction in the respective columns from right to left. (A number of students will go through the problem in the direction in which one reads, from left to right.) Beginning at the first

(one's) column, one does the problem 5 - 8 =?. Of course, (in the field of non-negative numbers) this is impossible, therefore one must make some adjustments. (Many students, as illustrated above, will simply do 8-5; they are not aware yet that subtraction is not a commutative process, i.e., $a-b \neq b-a$ when $a \neq b$.)

B. Because 8 is larger than 5, one must borrow from the next column to the left. This is done by subtracting one from the top number in the column, crossing that number out, and putting the number which is one smaller above it, then putting the one next to the 5. The problem now looks like:

15-8=7, therefore:

$$\begin{array}{r} 21 \\ 4\not{3}5 \\ -198 \\ \hline 7 \end{array}$$

C. One now looks at the next column to the left to see if the top number is smaller than the bottom number. If so, then one will need to borrow from the next column to the left. The same procedure is applied of subtracting one from the four, crossing out the four, replacing it with a three, and taking the one and putting it next to the two. Now one subtracts 9 from 12.

$$\begin{array}{r} 3\ 12\ \ 1 \\ \not{4}\ \not{3}\ \ 5 \\ -1\ \ 9\ \ 8 \\ \hline 3\ \ 7 \end{array}$$

D. Inspection of the next column to the left shows that the top number is larger than the bottom, therefore one can subtract in the normal way.

$$\begin{array}{r} 3\ 12\ \ 1 \\ \not{4}\ \not{3}\ \ 5 \\ -1\ \ 9\ \ 8 \\ \hline 2\ \ 3\ \ 7 \end{array}$$

Use of this method is amended for cases such as $\begin{array}{r} 100 \\ -99 \end{array}$, because trying to borrow from the column to the left is impossible if the column has a zero.

This method for subraction is fully adequate for most any problem involving subtraction. It only needs to be learned and followed rigorously. Notice too that it requires very little understanding of the process itself, which makes it appear "fool-proof". A teacher should simply teach her class the method, give them practice and the means for subtraction will have been taught.

Observation of children doing subtraction problems such as this indicate that, for some reason, this is not enough. To give this method for subtraction seems only to be giving a list of rules that can be broken or forgotten. And forgetfulness is a problem for teachers. They often complain that they taught such-and-such a short time ago, the kids were doing it well, and now they have forgotten everything we did. Or, they complain that their students should have had such-and-such last year; that they should not be needing to teach it. I suspect that previous teachers have taught these students what they should know, but that the students have, once again, forgotten. It appears that the children continue to forget until they have had enough time, in other words, until they have been re-taught a number of years in succession.

In summation, this method of teaching subtraction has two fundamental flaws. First, it depends on accurately recalling an ordered procedure. Many students will neglect certain critical steps, e.g., going from right to left, or they will adulterate steps to make it into a problem which they have seen and worked with before, e.g., 5-8 is equal to 3. The teacher is thus pressed to drill into her students not only a procedure but the importance of following a procedure rigorously, to the degree that subtraction is no longer a conceptual process, but now is merely a specified set of instructions. These children also have difficulty doing word problems, that is translating a real life situation into mathematical terms, such as addition and subtraction.

This seems a reasonable outcome if subtraction or addition are not understood as processes. The teacher I worked with in Kankakee was very desirous of help in teaching children how to do word or story-problems. She also was very distressed by her students' inaccurate application of the subtraction method, as was mentioned at the start of this section.

The second flaw of using this procedure is forgetfulness, which as just explained, is endemic among these children. It appears to me that the procedure is put into a short term memory track, and forgotten, as it is no longer called upon, that is, as the teacher moves on to multiplication and division and leaves three digit subtraction behind. Teachers are very frustrated by the degree of forgetfulness of their pupils.

Thus, when I am asked to help a teacher work with her children on three digit subtraction, or any other math task, I am aware that they are asking me for a method of instruction which is more effective than the present one they are using, being aware that the present method is full of flaws, as well as being very unexciting to the students.

An obstacle to teaching children - difficulty in maintaining full class participation and concentration

Besides the material to be taught, e.g., subtraction, there is the manner of teaching, how the teacher conveys the material to her students and what the students do in response to being given the material. One teacher I worked with had a great number of complaints in this latter area. She was not able to stand in front of her class, explain subtraction, give them work book problems for practice and go on to the next topic of multiplication after the predetermined amount of time. She was working with a group of students who were not docile, highly competetive with one another, and generally not cooperative in a manner that was necessary in order for her to teach as she wanted to. I believe this falls into the area of teaching called discipline. To discipline these children is not easy.

I proceed now with a description of the dynamics of the class's attitude and behavior as I experienced them when I taught.

First off, the children are not sitting straight in their chairs, eyes forward and alert, awaiting my instruction. Perhaps three out of twenty-two are, on a good day. Many are slouching in their chairs and waiting to hear what I have to say. Others are chatting or combing their hair. It is not an inviting or reassuring audience.

I begin the lesson by asking questions I expect everyone to know the answer to. This is to bring everyone to life and interested in listening to what I ask. I do this because they enjoy success greatly and want very much to show off what they know. They also exhibit a huge pleasure in being able to speak legally and be listened to. The teacher is usually the one who is allowed to speak and to be asked to speak is a privilege, just as is being asked to come to the blackboard or take a note to the office. Naturally,

the children relish these opportunities, as evidenced by the slow deliberateness they proceed with when doing these special activities, stretching the amount of time necessary for the activity to be as long as possible.

It is their desire to say the right answer outloud that makes them attentive to me and what I am saying. The unfortunate aspect of this ploy is that only one person can actually have the opportunity to respond, and the children will start shouting out the answers in order to be the first one. Thus I request or explain that shouting out is not allowed, that I call on the person who is allowed to speak. This has disadvantages; those who do not get called on to speak (this especially applies to the most active ones in the class) lose interest. I suspect that this is because they are losing their reason for listening and thinking if they are not able to show it off to me and the other students. When a student quits participating, my stomach literally stops functioning, because I am aware of the potential outcomes of this loss of participation and interest. First off, they are not involved in the development of the lesson, and thus may be missing the instruction and concepts that are preliminary to the final intention of the lesson. This means that finding their way back into the main stream of the lesson may be very difficult. Also, to not be working with the lesson is losing time, and often means that the teacher will need to individually instruct the child, which is not the primary means of instruction in a traditional classroom. Another problem can occur if the child enjoys talking and interacting with other students, for this is what the student will now begin to do. Some students are also very much in the awareness of the other students. What these "attended to" students say or do, directs the actions of the others, for them to stop their active participation in class is a powerful pull for others in the class away from the lesson.

The most subtle and undermining affect is just the change in atmosphere. A group of people focused upon one topic is much easier to direct, instruct or lead than a group who have divided themselves into smaller subgroups focused on other activities different from the lesson. The latter creates an atmosphere of dissipation and uncontrollable, sporadic fluctuations, that makes it very difficult to coordinate the class again. A few students can begin the flow towards a class of diverse activities simply by breaking up the full group focus that was established.

I emphasize that loss of a student's participation in the class activity is destructive to the education of the child individually and to the energy and enthusiasm of the class as a whole, no matter if the student is discreet about his lack of attentiveness or not.

The usual response to this deviant activity is disciplining by the teacher. The child is told to be quiet, to listen to the teacher, or is pointedly asked a math question when the teacher knows he/she is not listening, i.e., the tactic of embarrassment. Some students respond to disciplining more than others. Interestingly, those that are most disruptive are the least responsive, probably because they are yelled at so often that yelling and disciplining have little impact upon their behavior.

Needless to say, a teacher does not want to constantly be yelling at or telling her students what they should not be doing. It is wasted energy. It diverts the attention of the students' from the teacher to the one who is being unruly. Thus, it is usually energy spent by the teacher not related to the material to be taught. There also exists a subtle sense of despair, frustration and anger in a teacher when she recognizes that she is constantly fighting with her class. There is a recognition that children are not keen on listening to the method of subtraction or whatever else, and that they are subtly seeking a means for subverting the intentions of keeping them quiet and orderly and learning what the teacher wants them to know.

The underlying resistance and potential for havoc is a force the regular teacher and I were constantly fighting. And she was exhausted at the end of her full day and anticipating the last day of school. I was exhausted after my hour and a half.

I want to emphasize more how defeating and exhausting teaching can be if the children do not desire to learn what the teacher wants them to learn. They are unreceptive, uncooperative and apathetic to the quality of what they are doing and learning in school. The degree of these features just mentioned varies according to the student and the class as a whole, but the opposition a teacher must put forth depletes her energy, takes away time, and diverts the focus away from the actual material to be taught. For this reason, a teacher very much desires to be able to teach a skill such as subtraction in a manner that the students are receptive to and which is effective and efficient for the transmission of the material intended.

The class' regular teacher and a number of other experienced teachers I have talked to, complain that children are just not as disciplined as they used to be. They complain that parents do not care about the quality of their child's education any more. How accurate these statements are I cannot determine, and they really do not matter to me when I am teaching these "unruly" children. All that I care about is teaching them so that they want to learn, so they understand, and so they know what they are supposed to know in a way that is as free from rule memorization as possible. I mention the last point because of the extent of forgetfulness and the opposition and dis-interest of the children to following rules that have no basis in their experience. It does not matter to me, therefore, if television or Watergate has caused these problems, what matters to me, and to the other teachers, is, given the children as they are, how do you teach things like subtraction and multiplication in a manner that the children retain the knowledge, are not apathetic to the material being taught, and, most importantly, are not opposing the process of being taught.

A child's responsibility and desire to learn math in relation to what the teacher conceives math to be.

Before becoming involved in becoming a resource person in mathematics instruction for grade school teachers, I was primarily tutoring math to college students, predominantly Black and, of these, the ones that were having the most difficulty with courses such as college algebra or calculus. Working with these students, I was always trying to build their understanding of a process, such as, what it means when one is taking a derivative. Another key issue was recognizing similarities or patterns: for example, the similarities and differences between taking the derivative of $(x^3+ 2x+4)^3$ or $(\ln x)^3$. We would work on these problems together and together try to build some understanding of how to do them, the understanding being the recognition of how these problems can be attached with the formulas for derivatives, i.e., $\frac{d(u^n)}{dx} = n \cdot u^{n-1} \cdot \frac{du}{dx}$ or $\frac{d(\ln u)}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$

Once the student has accepted these rules as short cuts for taking a derivative (rather than doing each problem as a limit), the focus becomes helping the student recognize in the problem what formula applies, what is "u", what is "n", in this specific situation, etc.

Tutoring is a creative exercise in the sense that I am always seeking to find the illustrations or precise string of words which will communicate to the student what I perceive the subject matter to be, which is the combination of what I know, and what I have been taught. Very rarely though am I required to make the student want to understand, to make him/her want to know. Although math is rarely the students' favorite subject, they are at least motivated by the necessity of passing a required course with as good a grade as possible. They come to me ready to learn and listen and work.

I mention all of this because I found the fundamental difference between teaching college students and teaching primary grade children is their desire to accomplish and do what they are supposed to do. Put another way, a college student will attempt to learn what is required of them for the degree, or quits or is kicked out of school. The grade school children I worked with were not nearly so docile and receptive. They do not have the motivation of earning a degree or some definite goal obtainable through passing primary grades with all A's. In short, if what I am saying to these students or asking them to do is of no interest to them, they simply resist doing it. Of course, a teacher uses many tools for discipline, coercion, etc. Essentially, she restricts the student as in, "You do this or else..."; or she puts certain values as good, worthwhile, pleasing, etc., upon the student who does as the teacher instructs, and applies the opposite to those who do not. Many children are obedient to an adult's demand simply because that person is an adult. As was mentioned before though, it may be true that students are less disciplined today. Perhaps they are growing immune like the insects that are becoming less and less affected by the insecticides used upon them.

Whatever is happening, the children I worked with were very difficult to control and keep in order. Their regular teacher actually spent half of her time talking with me in expressing the frustration she felt through not being able to direct the class as she desired. She noted that in previous years she had used the command "shut up" perhaps once or twice, when the room became extremely unruly. This year, she was compelled to use it a lot more.

When I first began teaching the class, after having observed it a couple of times, I was literally frightened by these children. I could sense their short patience, their keen eye for when to make a deviant move, their high energy level and obstinancy to doing what they are told.

A quick reprimand or direction to pay attention was totally ineffectual. All of the previously mentioned means for bringing children into line, making them do as instructed, had had little effect upon these students, or, more probably, these means of discipline no longer had impact upon them. The only means for control I had was to keep them interested in and active with the lesson I was teaching - - and this was a very skillful and demanding task with twenty or more students to coordinate and keep interested together, on the subject I was presenting, no less.

At points during the semester, I would become resentful of this situation, that these children took so little responsibility for their learning. I felt I was responsible for everything: for their desire to learn, for their ability to stay focused on the material at hand, for their motivation to play with and interpret for themselves what was being presented - in other words, I began to feel totally responsible for their learning.

Maintaining their interest and positive participation in a math lesson for one and a half hours was not easy, if only because of the length of time. I also had difficulty in working around their shortness of patience. For example, if they did not understand what I was doing or talking about, they would become resentful and turn away from what I am doing. If they answer a question out loud in class, which turns out to be wrong, they become frustrated - - they do not want to be wrong and especially, to have other people know it. If I do not call on some students enough, they will quit participating - - they are not receiving immediate attention and recognition for what they are accomplishing. These students also, especially at the beginning of my work with them, would jump to answers that, somehow to them, held the potential of being right. I believe they found it extremely important to be the first person with the right answer and also very important to make sure that everyone is aware that they were first. To my dismay, too little thought was being given to the questions I was asking - - their responses

are closer to speculations on the futures markets. I realize though that there exists a common misconception of math, that it is a subject of the right and wrong where some are gifted and see the right whereas most are not. I perceived that these students were working under this conception of math, for I infrequently would sense that they were evaluating a problem or attempting to understand something that perplexed them. The first time I had a student yell at me, "I don't get it", and tilt his head to the side, I was elated. I then felt that he recognized both what he did not know and that perhaps with another explanation, he could begin to understand. This is a quantum leap as far as a child's ability to learn in a conscious and selective manner, rather than purely as a spontaneous response.

Thus, even if I had managed to gain the interest of the class, it could quickly be lost. As mentioned before, given the tendency of these students to direct themselves irregardless of what they should or should not be doing, and my desire to use as little extrinsic coercion as possible, I realized that what I required was the means for keeping them active and attentive through a desire on their own part to do so.

An essential difference between my outlook on teaching math and the outlook of most teachers is that I perceive math as an exciting and creative subject, thus I believe that it can be stimulating for primary grade children as well. A teacher who sees math as a bit of a drudgery, a set of rules to be memorized, and, created and understood by some superior intelligences only, is unable to recognize the possibility of it being a stimulating subject to teach. If she is further burdened with a group of students, as just explained, who feel little value in learning what does not interest them, she becomes essentially trapped, just as the teacher I was working with felt herself to be.

I recall her surprise after the first lessons, when his teacher asked me, "How come they didn't teach it to us this way?" I believe "they" were her instructors in college, preparing her to teach math in grade school. Sometimes, though, I thought "they" meant her teachers from grade school. In any case, an accurate reinterpretation of the question would be, "How come no one ever showed it to me like this before?" What were ordinarily the generator of tasks and drills, such as multiplication and division, were now becoming concepts of and about themselves, and what is more, concepts of interest and wide applicability. The formula $20 \div 4=5$ has very little use, except if one knows the relation of multiplication to division, one can state $4x5=20$ and then $20\div5=4$. All in all though, the universality of $20\div4=5$ is relatively small, e.g., it does not help much with $6\div3=?$. The teacher I worked with said that teaching division was one of the hardest things for her and that her students had a great deal of difficulty in understanding or doing division problems. I am not sure if she felt teaching division was difficult because her students did not do well with division or, if division was a subject that she did not have good ideas about, to teach, no matter how easily the children took to it. In any case though, I presented division to the class not as dependent upon multiplication, but as a concept unto itself.

Often division is essentially taught as follows: Given the problem $20\div4=?$, one reinterprets it as $4x?=20$. (Notice how difficult this process becomes if one has $21\div4=?$ reinterpreted as $4x?=21$). The approach I used, which Hal Taylor discovered as effective, was to consider division as breaking a whole into parts, each part of equal size. For example, given $20\div4=?$, I talked about the problem as, "you've got 20 and it says to divide into four parts." So I have this whole thing (I draw a box on the blackboard) and I am dividing into four sections. (I then divide it into fourths, each section of relatively equal area.) Now I've got 20 here. How much should I put into each sub-box so that I have 20 all together. Should I put 1 in each box? (I draw a 1 in each of the four sub-boxes.")

"No, that's too small."

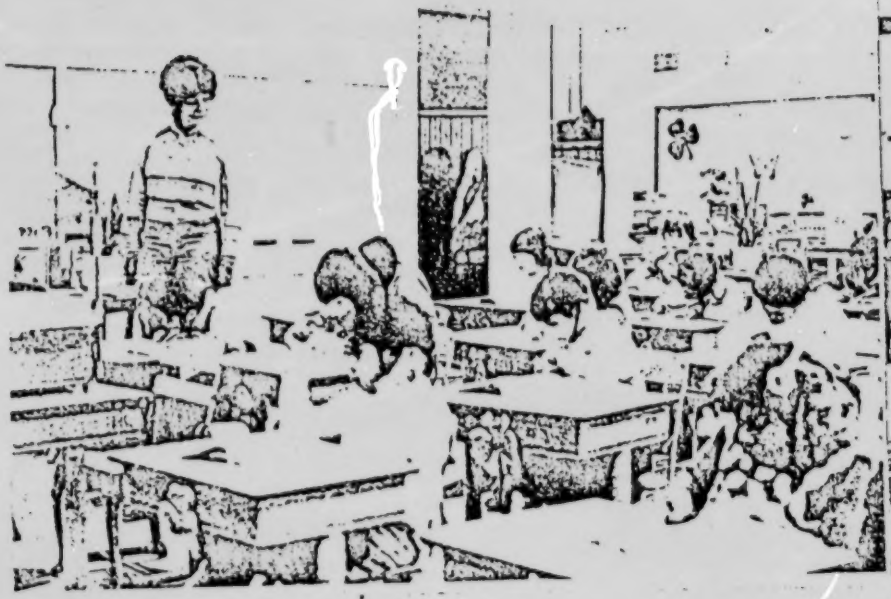
"Perhaps I should put 10 in each box. (I draw a 10 in the four boxes.) How much do I have altogether then?"

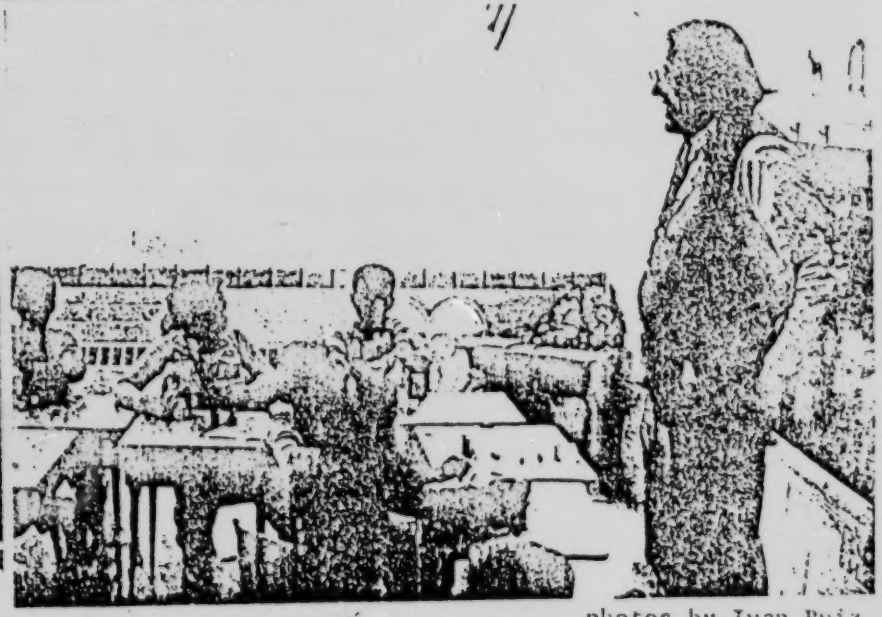
And we keep on going, bringing the estimation closer to home, i.e., closer to five in each box. Notice how much more useful this concept of division is. For one thing, it stands on its own, although it can easily be related to multiplication, which we did later. Also, it does not depend upon knowing formulas. Suppose one gave the problem 225÷5=?. Rarely do multiplication tables go as high as 15, so using that method is not useful here. One can experiment though, and see how much is needed in each of five boxes now, to make a whole equal to 225.

When a teacher asks me how come no one ever taught her this, it says two things to me: 1. she has not yet fully conceived division as literally breaking a whole into parts and 2. that she probably did not even believe that mathematics can be understood in a manner which relates to her experiences, e.g., that of breaking up something whole. She has not yet found the means by which mathematics is made to "feel right", that is as believable and experiencable phenomena to which one can say, "Yes, I agree as well." If a teacher has not experienced math as such, she cannot be expected to teach it as such. If a teacher "knows" division to be that sign which indicates referring to multiplication and/or some formula, this is how she will teach it to her children and most important, recognize its acquisition. That is, when it becomes apparent to the teacher that a student is doing and working with division as she has experienced it to be, she says the child "knows". Her motive and her idea of what division is are very similar to her criterion by which to judge whether her student knows division. If the student is experiencing or knowing division in a different manner than the teacher has thus far experienced or been aware of herself, she is highly unlikely to recognize that her student is beginning to understand division. Knowing what one's students know, i.e., at what stage they are at, is critical to effective

teaching, so this is no small issue. Not having an awareness of or a knowledge of alternative approaches to division restricts the teacher's ability to alter how she is teaching division. It also restricts her ability to recognize the growth of the knowledge of division in her pupils.

Perhaps a teacher should not need to have someone show her that division can be conceptualized as breaking up or dividing a whole into a specified number of parts. Perhaps she should be discovering this by herself. But, if she does not believe that math can be interpreted by her or that it can be made "sensible" i.e., that math is more than rules and formulas, she has little motivation or reason for attempting to pursue the issue any further. I perceive this as similar to a situation in which a person is in need of something yet has no idea what this something is, or that it exists and where to begin looking for it. For this reason, a teacher is restricted in finding alternative means for teaching math because she does not perceive math to be anything but what she is already teaching, as much and as urgently as she wants to find this "better way."





photos by Ivan Ruiz

Classroom Etiquette

Harold Taylor

As a resource teacher for Lincoln Consolidated Schools in Ypsilanti, Michigan, it is my role on a daily basis to go into classrooms and teach math to other teachers' classes. I do this in any grade from K-6. I have done this for the past five years. My background is that of an elementary school teacher (16 years) interested in math but holding a Master's degree (M.A.T.) split in hours exactly half in reading and half in math. Therefore, I am a reading diagnostician and math specialist. I have pursued Doctoral work in Elementary Education at the University of Illinois and was involved in Max Beberman's Washington School project in 1969-72.

For this project I have been employed as a resource teacher consultant at Kankakee in the Stueben Primary Center. I did work at Stueben that exactly paralleled the kind of work I do at Lincoln Schools. I would arrive on Friday morning and be assigned a schedule to work with teachers in various grades introducing or following-up a variety of subjects at the teacher's request. Follow-up sessions were undertaken throughout the rest of the month by other resource persons (a professor or one of his assistants) and preserving continuity of ideas remained primarily within their hands. Since I was a "practioner" it was my job to put into effect as much of the "theories" about problem solving or logical development in subject matter as I could, given the one-period-per-month-per-teacher schedule I was following.

I use the word "theories" very loosely, as some of these "theories" are my own ideas about classroom management, children's thought processes, logical subject matter development and personality interactions, where teachers, children, and systems of teachers and children are concerned. I also use the

word "theories" to refer to the ideas of the staff of the "Committee on Culture and Cognition" and the research presently or historically connected with the "Committee", effecting either my work or the work of other teachers.

Etiquette

Operating within the confines of an elementary classroom requires first and foremost what I shall call "Classroom Etiquette." In ordinary terms the idea of "control" simply doesn't fit. "Control" implies a unilateral application, "etiquette", on the other hand, at least implies the "bi-lateral" circumstance that actually is education. "Etiquette" within the classroom more adequately defines the atmosphere necessary for the pursuit of academic subjects and further hints at some of the qualities the children who have that "etiquette" must also possess. These qualities are personality traits that teachers, and more importantly parents, should strive to instill within their children -- those of self-discipline, composure, and courage. These are the traits that teachers must develop simultaneously within each individual whilst they are also teaching them to think. To assume, in this day and age, that they already exist in any child, but especially elementary school children, is to ignore the reality of poorly trained parents and teachers, not to mention the narcicism of the present psychology at work in society.

Self discipline -- can be defined as learning to do things you don't want to do when you least want to do them.

This precisely is the only freedom children will ever have, with it they can do anything, without it, nothing.

Composure -- can best be defined as "Do not disturb the Wa." (In the book Shogun "Wa" refers to the spirit or psychological climate, the peacefulness of things.)

To remain composed allows the individual the climate in which to think; if he is disturbed or rattled, then he isn't thinking.

> Courage -- that firmness of spirit, the confidence that encourages and sustains, the mental and moral strength enabling one to venture, persevere and withstand fear or difficulty firmly and resolutely.

Any elementary teacher having taken courses in higher math can tell you about fear and the courage required to sustain him or her.

Classroom Etiquette

Etiquette is a set of rules used to govern behavior. There are certain qualities about etiquette that are easy to describe. In everyday greetings, the polite "hello", or "how do you do?", echo the individual's desire on the one hand to belong and simultaneously to be recognized. By involving himself in the ritual of politeness, he admits his desire to belong. By belonging in appropriate ways, he wishes to be recognized. This delicate balancing represents a subtle form of "fishing and queing" that all of us engage in, and our ability to do so ranks us among our peers whether we like it or not, and whether we participate or not.

The reasons for certain form of etiquette seem obvious as the logic for them is obvious. Taking turns provides for good order in whatever is being done. But some etiquette forms are followed today out of habit although they started with logic many years or centuries ago, and the logic for them has long since disappeared. The handshake, for example, comes from the meeting of men in the middle ages. The right hand (sword hand) was extended in a show of friendship to indicate the intent not to do battle. Today we make no such connection but carry on the tradition by habitual good nature.

Classroom etiquette is established very early by teachers, as this is the means by which a "mob" is turned into a viable group, congenial and cooperative enough to provide the atmosphere so very necessary for the

bilateral aspect of communication required of all individuals and groups who wish to participate in the educational experience.

The classroom can be, and usually is, filled with several different etiquettes at once. It is for the teacher as leader to help formulate a single etiquette that will produce the climate necessary for thinking. If the intent is control, the point of etiquette is missed. The "control" is not the primary concern but rather it is secondary to the concept of developing an atmosphere conducive to questioning and thinking. The rules for operating a classroom must be such that they focus on the conditions of the dialogue not the behavior per se. The children and teacher should be conducting a conversation over the subject matter, not over the behavior of any particular individual in the group.

Children frequently have an etiquette of their own. Although esoteric in nature, it is none the less this etiquette to which many children ascribe while they seemingly attempt to disrupt the class. They are simply gaining points -- recognition in the eyes of their peers in terms of that etiquette.

The etiquette that I strive to produce is one that focuses upon the conversations that can be held by a large group that would be guided by me. The rules are mainly those of conversational etiquette.

1. Speak in your turn.

2. Raise your hand to be recognized. (The teacher must be judicious here about calling on people.)

3. Be patient, allow others plenty of time and quiet to figure things out. (The teacher must learn patience also and <u>give</u> time to everyone who can participate.)

4. The teacher must couch her questions in a manner that:

   a. reduces or eliminates the potential embarrassment of asking a question;

b. allows children a chance to join in a group response by using questions like "How many had answer X?" or by accepting several solutions then proving one of them correct so that thinking used by children can be checked by children;

c. provides students with an opportunity to hold their own ideas in secret, allowing others to come to similar conclusions about the mathematics being studied.

All of this serves to develop an atmosphere inside which children can invent arithmetic. It is a method that allows the group and all the individuals in it an opportunity to focus their massed intelligence upon problems and patterns so that they might develop a clearer understanding of arithmetic by having deduced for themselves the concepts that are put before them. The teacher's role as guide is to place before the students various patterns and examples that will allow the students to develop ideas about the examples that will lead, through deductions, to conclusions that will ultimately be used by students to produce predictable results in other similar problems.

The rules for such interaction can easily be ascertained, but the spirit of this etiquette remains a quality that is essentially ineffable. Ineffability here refers to the fact that a particular item cannot be described because prose will not adequately deal with it. Our senses will deal with it but our words fall short of the desired intent. (For instance, the taste of strawberries is not easily or adequately described by words -- better you should eat one.)

So this is also true of the spirit in which the etiquette is developed. This is not something one discusses or lays down to rules, rather it is "how one acts." It is the personal carriage of the individual, his manner, or the quality of interpersonal relations as pursued by him or her.

There is a careful balance necessary, and that balance cannot be maintained by anyone who does not have a feel for the components that make up the etiquette that is the spirit of the classroom.

This is all very spontaneous. An elementary classroom is not the playing of a well rehearsed symphony, it is rather jazz -- a jam session. The teacher must be the leader and know where and how to get the most from each "player". Each "player" must know when to play, either as soloist or back-up, but in all regards never at anyone else's expense and certainly never at the expense of the group.

Classroom Etiquette and the Resource Person

Imagine for a moment the difficulty that all this places on the delicate relationship that exists between resource persons and classroom teachers. What happens for instance when they are ... "not looking at the same classroom"? What happens when the resource person is not sensitive to the "etiquette" established by the teacher, and the resource person "disturbs the Wa"? What happens when the classroom teacher has little or no developed etiquette within the classroom and the Wa necessary for thinking as a group is already destroyed, and the resource person perceives this? What happens when the resource person doesn't perceive this?

It seems that we have several possibilities for resource persons that need to be considered. Some of these are displayed in the chart below.[1]

[1]This matrix needs expanding but for now, it seems as a starting point in our inquiry.

Resource Person

Teachers	Perceives Etiquette	Does Not Perceive Etiquette
Sufficient Etiqette	A. Best possible working relations are here.	B. Resource person will be of little use to teacher.
Insufficient Etiquette	C. Resource person can be of assistance.	D. In the dark here.

Resource people need to avoid "B" and "D" at all costs for, in the one instance, they will be of little use and in the other they will be left in the dark. Of all the possibilities, "B" is the worst for it carries with it a quality on the part of the resource person that will eliminate any interaction at all. Having misjudged the situation, poor decisions will be made and non-involvement might occur, or worse, bad involvement might occur, by jumping into a perfectly viable situation and turning it into chaos.

Comparative Etiquette

There were three classroom situations this year, in which I had to function, that were for me good examples of observing or not observing the ettiquette at work (or perceiving or not perceiving the presence of etiquettes within various classrooms). I'll number these classes in Roman numerals as I, II, and III. I'll further subscript them using the letter from the matrix on top of this so; we have I_C, II_B, AND III_B. Note that here exist two type "B" interactions, they will be as it turns out, totally different conditions because the reasons for the perceptions changed for each.

Resource or research personnel do not ordinarily come into a classroom and help teachers solve their problems. Rather classroom teachers are usually asked to help researchers with their problems. So, going into these

classrooms we had everything rearranged. We had researchers working on particular problems, resource people working as partners to the teachers, and demonstration teachers helping give lessons to the teacher and the children. In all of this then, my job was to come in and demonstrate techniques for advancing arithmetic knowledge couched in problem solving techniques that were closely tied to an understanding of both the etiquette required as well as the continuity of the subject matter.

The three cases are described below.

Classroom I_C

Classroom I_C is referred to as a "C" classroom because I perceived that a large portion of the problems in that classroom stemmed from classroom etiquette and also the fact that several "other" etiquettes were at work.

The teacher of this class was very happy to get all the help she could. She cooperated in every way. A resource team "ganged up" on her room. Many times there were as many as four adults in that room helping to teach subject matter or etiquette, or both.

The teacher complained that everything being done from her texts left much to be desired. The children were unruly and the subject matter presentations seemed useless, especially since the text introduced a new idea about every 7-10 working days. This she admitted was confusing and left her with an empty feeling that nothing seemed to be "tied together."

Our plan of attack included every aspect of resource involvement. I would always make a point of going into her room when I came to Kankakee (this was a once a month visit) and others would come in on a weekly basis to do several things; either remove individual children or small groups or remain within the room as a partner in helping develop academic and social strategies to improve the spirit and direction of the room.

Upon observing this classroom we felt that, more than any other, this one would require our strongest and most diligent effort in order to relieve this teacher's feelings of anxiety and her lack of confidence. She needed someone to help her pull loose ends together and start making sense of what was going on. She needed analysis of various individuals within her class and recommendations about how to attack solving the academic needs of her most frustrating pupils.

I had spotted the three serious "problem boys" as they walked into the class. I took them aside and made a point of learning their names and saw to it that they knew mine. Further, I made a point of making physical (friendly) contact with each. I placed each close to an adult (since there were four in the room) with instructions to the adults to keep these kids "on task" and help them think through the situation.

The teacher had asked for a lesson on subtraction with borrowing. I had to discover the class' ability in addition first, so that I could sort out the problems in basic facts from those of logic and place value.

I explained the etiquette. We would stick to the rules for group conversation, speaking each in his own turn and only when called upon, that we would have patience with each other for time enough to think.

I started out with doubles, and showed the children how to use them to figure out more complex problems. I showed them ho to use addends for the sum of ten and how to turn problems into tens plus. This procedure was branched off into two directions, one direction was two-digit addition with and without carrying, the other, basic subtraction, and subsequently subtraction with borrowing (see chapter IV.).

These areas were covered quickly, as third graders in February are able to follow this instruction quite well. The etiquette and spirit in which

this was accomplished developed from the fact that I had ascertained quickly what the personality problems were and who they were, and made an early effort to stave off having to deal with them while pursuing the ideas we would encounter. Further, it was imperative that these boys got the attention they needed, make some "correct" responses through proper questioning, and experienced the real success of finding out for themselves answers to questions they needed to have anwered.

I worked through the lesson carefully, curbing any attempt to breach the established etiquette. I tried to involve even the most reluctant child, taking cues from the other resource persons in an effort to involve our "special cases". During the lesson I tried to show the teacher where her problems (both academic and classroom etiquette) really were, and how certain weaknesses in addition, place value, and logic were the causes of most of her children's problems in subtraction with borrowing.

The teacher had a lesson in developing and using etiquette to set up and carry off a lesson. Further, she had seen (possibly for the first time) that there is a method to connecting what is known to what needs to be figured out in such a way as to make it very clear and easy to follow.

Classroom II_B

In case II_B, my perception of the actual classroom etiquette was mistaken, the teacher's perception of the etiquette needed was ill conceived, and neither of us knew of our own misperceptions. I was asked to demonstrate teaching the subtraction algorithm.

About ten minutes into the lesson I began, to my horror, to discover that the etiquette I thought ought to be there wasn't. On one occasion the teacher jumped in and saved me by turning a child around (he'd turned his back on me) and getting him back into the mainstream of things. Without a complete feeling

for the spirit I thought was there the class came apart. In my usual approach of getting children to participate within the logical framework, figuring things out as they go, their excitement rises higher and higher. I can control this excitement only if the proper etiquette is established. In this case it was not and slowly their excitement began to turn to chaos.

Here is a classic case of the resource person -- me -- not having ascertained accurately what the classroom etiquette was. I didn't know the system used by the teacher, because I hadn't had the opportunity to talk to the teacher before I began. The ettiquette used by the teacher was one in which she maintained control over the entire class while one child at the board thought an idea through. This was a "one-child-at-a-time" affair, and everyone else waited until that individual finished his step-by-step procedure to the end.

Without knowing this, I was demanding from the children the exact opposite--that is, every child examining one idea and trying to share in the solution. They did not understand the etiquette for this kind of teaching and they began to compete with each other. In the excitement being generated by the lesson along with their lack of etiquette for the situation, chaos arose, and I began to resort to controlling the class instead of the lesson.

In discussing the results of this fiasco with the teacher, I discovered that we had both misjudged the situation completely. The teacher indicated that "she would work on the problem" and in the subsequent lesson it was obvious that she had zeroed in on the difficulty (the ettiquete) and solved it. The next lesson went beautifully and I felt it was worthwhile to everyone.

There is however a quality here that needs to be investigated. The teacher didn't know me or how I operated and she had little time to prepare herself or the children for the first lesson. As resource people, we thought we knew the situation well enough so that we could slip in, do a demonstration, and slip out.

All this added together (fortunately) in such a way as to provide us with a new insight into each others' ideas about education. The teacher changed a little and we changed a lot.

Classroom IIIB

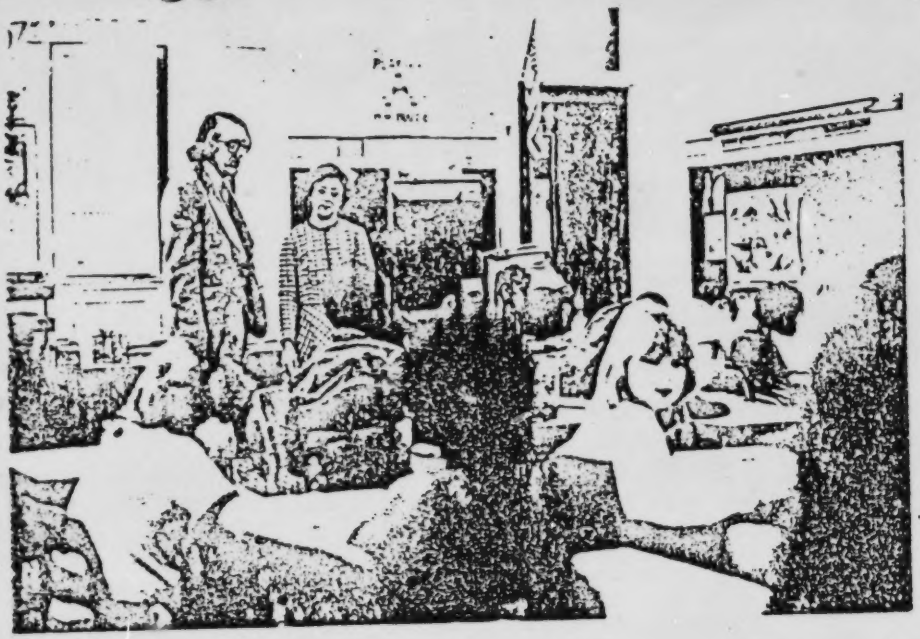
In this case we didn't, at first, think we'd get into the classroom for demonstrations at all. The class was well ordered, held in tight control, quiet to the point of being unique within the school. The teacher seemed distant and hard to get to know. Our conjecture was that this teacher held the children reined in tight by an etiquette of strong controls and that her apparently distant personality was what the children were responding to. When I took the class over for a demonstration, I expected it to fall apart instantly in the atmosphere of massed questioning that I was about to pursue.

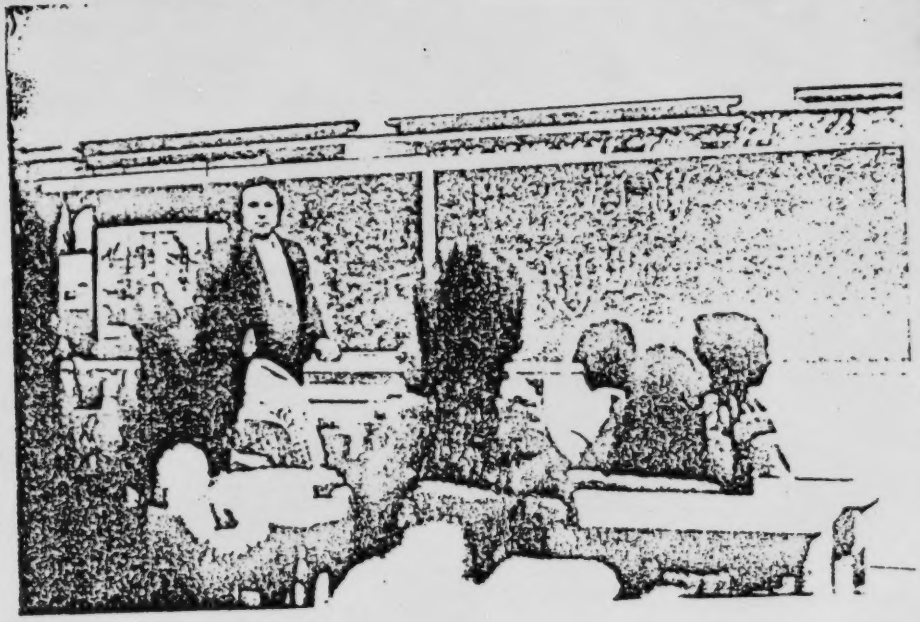
Nothing could have been further from the mark. Our picture of the reasons for the children's behavior were totally unfounded and poorly derived. As I began the lesson the children responded beautifully to my questions. They eagerly participated, trying very hard to draw conclusions from the patterns put before them without engaging or competing with one another, and giving plenty of time and space to each other for thinking and reacting. Much to my surprise, when slower children (purposely called on because I knew they had the answers or the teacher pointed them out because she knew they did) gave the correct answers, the class would burst into spontaneous applause in support and appreciation for their efforts. The lesson drew them further and further along and, all the while, only the most excited of them would ever breach the etiquette with a blurting out of information that they could no longer contain. This was controlled with a mock look of disbelief on the part of the teacher which quickly brought the child's attention to this lack of discretion and, with a knowing smile, they would contain themselves and join in again instantly.

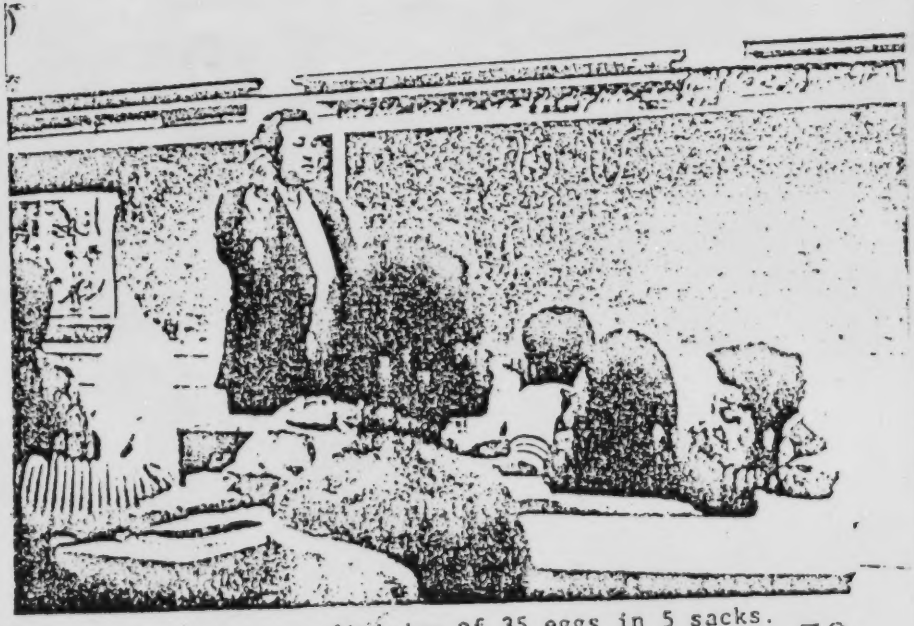
All the while, the teacher worked her quiet magic on the slowest or shyest of them, encouraging them into the mainstream of thought and action, and reinforcing their confidence in themselves.

We readjusted our naive assumptions at the end of this rather unique experience, this "happening", and reevaluated our abilities to judge, having learned a lesson in perception that will not soon be forgotten. Here was a teacher's teacher, one of those magicians whose personal qualities are exemplified in her students' great love to learn and their fondness for their teacher. What we thought was being demanded was being given, because the children held her in high regard. We found under the outward appearance of a "distant" person a true child advocate, a teacher who loves her children and from that love derives the most each child has to give, and one meter more.

We shared a great deal in that class. The respect that developed between all of us produced, in our final day, what I can only call a jam session. Three resource persons took turns going before the class and doing something slightly different with an idea about equations and sequences. But on this one you'd have had to be there, because we didn't have any poets to record what went on, and only a poet could have captured that event adequately. (See transcript in Chapter X.)







49 Easley checks the teacher's reaction while Taylor works with the class on division of 35 eggs in 5 sacks. 50

Primary School Mathematics

Harold Taylor

Teaching the Mathematics

As a demonstration teacher for grades K-6, my view of mathematics must necessarily encompass all of the grades that I teach. For this project, and for the benefit of classroom teachers, I have started developing a system of flow charts that ties together the concepts of arithmetic that I feel are vital to understanding the types of problems children have, and which can be used to help teachers decide what to do about those problems.

There are essentially three tracks to this program: computation, place value, and logic. All of these areas are presented to children in a problem solving posture. Each area relies heavily on the use of hardware such as Cuisenaire Rods, or Multi-base Blocks or Attribute Blocks, etc., for presentation to the entire class or small groups. Any other presentation made to children is made to the entire class by the teacher from the chalk board without reference to paper and pencil work. Such activities may or may not require the use of models like rods etc., but the idea is to present children with a pattern from which they can, by inference, draw conclusions about numbers in such a way that they can then be applied by the children in other situations in which they may encounter similar types of problems.

Each of the three tracks can be undertaken by itself or simultaneously with the others. The activities will be usable with various textbook series, although the method is totally different from a textbook approach. The heart of the idea of the presentation is the use of patterns and single or double step logic to help the child invent arithmetic. Very frequently the method will rely totally on the massed intelligence of the class to observe some event

occurring or some pattern unfolding so they can grasp the idea these events or patterns represent and predict outcomes in similar problems that are later presented to them. The idea is to get a firm grasp of the mathematics mentally and/or physically, to gain an intuitive feel for the math long before any computation in written form is undertaken.

In the charts appended, the progression follows, for the most part, the usual path, addition, subtraction, etc., but note that the emphasis on counting as a means to these ends is supressed.

The place value system is pursued through the M.A.B. or into chip trading materials with emphasis on games and activities that produce an insight into place value that produces very strong understanding even in first graders.

Addition remains the key to literally all other basic operations, but developing addition is done via Cuisenaire Rods and M.A.B. (multi-base-arithmetic-blocks), not individual counters.

Logic begins with attribute blocks, developing the simple sortings of attributes that lead into more sophisticated studies of the relations that exist between the attributes. Later the work in logic develops into implications and inferences (if-then) via matrices and a system of problems called Mind Benders. This is applied to a variety of other problem solving situations also includes analysis in reading.

The Presentation

Ordinarily I will take the entire class and, depending on the age of the children and the subject the teacher wants me to pursue, I work from the front of the class and develop a lesson en-mass.

For instance, if I were to develop a lesson on fractions it might go something like this (assume a fifth grade class):

1. I check for multiplication skills if they are strong...

2. I develop fractional parts of sets either with Cuisenaire Rods or with colored chalk, as follows.

3. With Cuisenaire Rods, I develop stair cases where white = 1, red = 1, light green = 1, purple = 1, yellow = 1, etc.

4. Then return to each case greater than w = 1 to establish number values for rods left out of the counting sequence. Therefore, when p = 1, w = 1/4, r = 2/4 or 1/2, etc.

5. However, in each case, I also establish that one is equivalent to, in the purple set, 4/4, in the yellow set 5/5, etc.

6. Now we have or can establish the equivalency of "one". That is the equivalent of one to 4/4, 5/5 etc.

7. Once this is understood, other equivalence classes can be investigated because all of them can be developed from this original class.

8. So 1/3 = 2/6, 3/9, 4/12/ 5/15...and 1/4 = 2/8, 3/12, 4/16,. 5/20 etc. This is because 1/3 x 2/2 = 2/6, and 1/3 x 3/3 = 3/9, and so on.

9. These are left on the board along with several other facts.

10. Then the rules for adding are introduced. Rule #1 -- you may add only fractions from the same class, i.e., the denominators must be the same. Rule #2 -- only the numerators are added.

11. Next step -- practice.

12. Now we attempt to add 1/3 + 1/4. This breaks Rule #1, BUT...

13. A new idea is introduced. We know that 1/3 = 2/6, 3/9, 4/12, 5/15, but we need to get used to the idea that 3/9 is one of many nicknames for 1/3 and that 3/9 is really 1/3.

14. Then we re-examine our problem 1/3 + 1/4 and discover from the sequence on the board that we can add the nicknames 4/12 and 3/12.

15. Now we add the fractions like this:

$$\begin{array}{r} (4/4) \times 1/3 = 4/12 \\ (3/3) \times 1/4 = 3/12 \\ \hline 7/12 \end{array}$$

The 1/3 becomes 4/12 because originally we multiplied 1/3 by 4/4 to get 4/12 and also the same is true for 1/4 x 3/3 = 3/12. It is important and easy to note that we are multiplying each fraction by "one" to accomplish our task.

16. We erase the extra work above the sum line. So we now have

$$\begin{array}{r} 1/3 \\ +1/4 \\ \hline 7/12 \end{array}$$

and the children are now asked to try to find a way to combine the numbers to get the 7 and a way to get the 12.

The etiquette here is very important because no-one should help anyone else with this idea. It should be deduced by each person.

17. A new problem is introduced, similar to the first (1/4 + 1/5). Now the answer is called for first. Some have it, some do not. We collect three answers and then, using our multiply-by-one rule, work the problem out to ascertain the correct answer.

18. A new problem is introduced (after time to think).

$$\begin{array}{r} 1/4 \\ +1/7 \\ \hline 11/28 \end{array}$$

This problem continues until a majority of the class has discovered a system that works. Then one numerator is changed to a number greater than one. Their rule may break down but the problem is worked out and a new system is tried by each until again a majority has a system working. Then

both numerators are changed to numbers greater than one, and new examples are introduced for children to work their "magic" on until they are secure in some method for solving these problems.

18. This procedure can be pursued out to the level of $A/B + C/D = \frac{(A \cdot D)+(B \cdot C)}{(B \cdot D)}$ given that the examples are carefully controlled by the teacher. The rule for control of course is: keep the denominators relatively prime and, at first, the numerators equal to 1.

Please note that throughout the presentation, every effort is made to allow the examples to do the talking. Therefore, great care must be taken to control the examples in such a way as to allow conclusions to be drawn from manipulating them.

Only after much play (this could be days or weeks) about in this way, does the instructor explain in detail the rules at work within the examples, thus formalizing the algorithms because only then can he expect the conversation to have followed the thinking of the children. In other methods, children are told what to do or left on their own in an activity, they may be allowed to discover something, and having done so the opportunity to capitalize on the discovery may have passed without the teacher ever being aware of it.

In the method I have outlined, greater care and control of intellectual inquiry is maintained, as compared to the alternative of small groups or individualized instruction. However, more patience and teacher involvement is required in order to achieve the high state of thought that is possible within the confines of this method. The above statement is not meant to detract from small group or individualized instruction. Rather, I wish to point out that there is weakness in all forms of instruction and that various methods should be pursued in order that these weaknesses be compensated for.

ify the game.

To up-grade this game, two possibilities come to mind.

M_1 The first is a modification of the dice.

$M_{1.1}$ 1 A single die is used to start the game. This can be changed at an appropriate time to 2 dice. (Changing the game to 2 dice is required of games in excess of base 5.)

$M_{1.2}$ 2 The die can be numerated, or one regular die and one numerated die can be used together.

$M_{1.3}$ 3 The die can become a 2 or 3 place die.

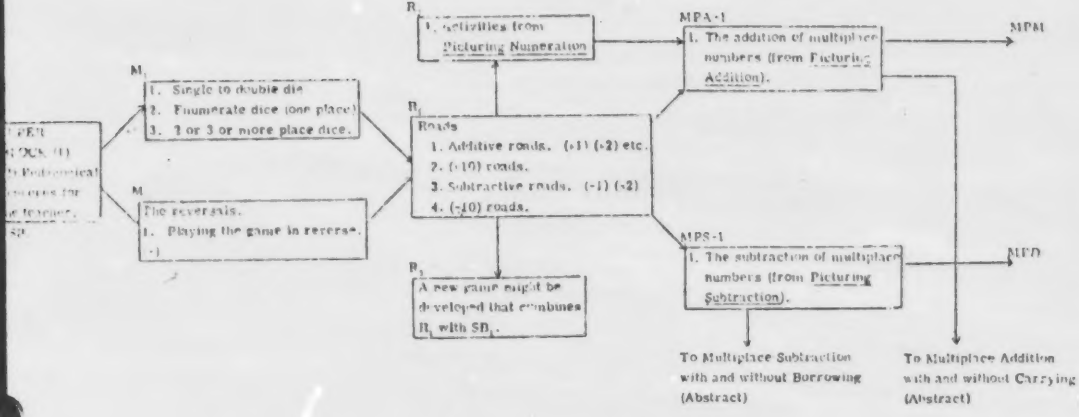
M_2 Play the game in reverse.

Now the game is played starting with 1 of the x' or x' blocks. The die is rolled and x' pieces are given away until all the total of x') are given away. The first person to do so is the winner. In this way, the one-for-many idea of borrowing is introduced.

$M_{2.1}$ At the appropriate time, dice modification ($M_{1.1}$) ($M_{1.2}$ or ($M_{1.3}$) can be instituted.

The Roads

Chip Trading


R1
1. Activities from Picturing Numeration
MPA-1
1. The addition of multiplace numbers (from Picturing Addition).
MPM
M1
1. Single to double die
2. Enumerate dice (one place)
3. 2 or 3 or more place dice.
R1
Roads
1. Additive roads. (+1) (+2) etc.
2. (+10) roads.
3. Subtractive roads. (-1) (-2)
4. (-10) roads.
M2
The reversals.
1. Playing the game in reverse.
-1
R3
A new game might be developed that combines R1 with SD1.
MPS-1
1. The subtraction of multiplace numbers (from Picturing Subtraction).
MPD
To Multiplace Subtraction with and without Borrowing (Abstract)
To Multiplace Addition with and without Carrying (Abstract)

Whole Related to Parts

47

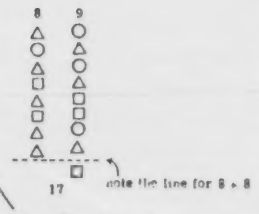
Now also

17 is related to { 9 and 8, 10 and 7, 11 and 6, etc.

16 is related to { 8 and 8, 7 and 9, 10 and 6, etc.

15 is related to { 7 and 8, 8 and 9, 5 and 10, etc.

14 is related to { 7 and 7, 8 and 6, 9 and 5, 10 and 4, etc.

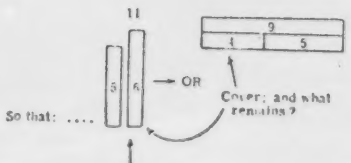


8 7 5 9
+9 +6 +8 +8

7 6 7 5
+9 +8 +5 +8

Practiced daily.

Therefore, making the set "17", either as individual units or with Cuisenaire Rods, can be used to introduce the relationship of the numbers in a subtractive way.



Final Complex Choices

At this point, children need to learn to make DECISIONS. Some problems can be solved by using either of 2 doubles or 2 sets of Addends for Ten or some less complex series of choices.

DECIDE:

What double?

8 / +8 is related to +__. Therefore, the answer to 7 + 8 = ?

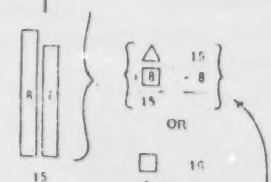
8 / +3 is related to +__. Therefore, the answer to 9 + 3 = ?

8 / +5 is related to +__. Therefore, the answer to 8 + 5 = ?

Note: By now there are several choices here, and any reasoned choice could be acceptable.

The major objective here has been to have children utilize 4 major logical structures to figure out and ultimately master the 100 basic facts for addition. To this end problems should be pursued at random on the board at this time while encouraging children to use these methods to help solve problems they have trouble with.

If the set 8 is covered it can plainly be seen that the set 9 is left or vice versa.



If the seven is covered it is plain that the 8 remains and vice versa.

Associations and Commutations:

1 1 3 2 3 3
2 3 2 3 3 1
3 2 1 1 2 2

2 2 4 4 3 3
3 4 2 3 2 4
4 3 3 2 4 2

Decide which group is easier for you to do? Why?

Footnote the Laws:

To Place Value Work

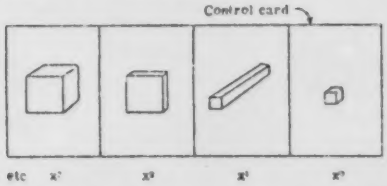
*These special cases can be used to help solve closely related problems. That is, since 7 + 7 = 14 and 7 + 8 = 15 are related, then, their subtractive inverses are related in the same way.

K or 1

PLACE VALUE--M.A.B.

(SB_1)

1. Introduction to base 3--Super-block using base 3 rectangular blocks.



The major objective in playing superblock is to acquaint children with the process of trading "ones" for "a Rod" and "a Rod" for "ones". The procedure for the game requires close monitoring and should be started with "1" group of children the teacher can trust to keep to the rules. The children take turns playing the game--rolling the die, collecting the ones; trading for Rods; and when they can, trading the Rods for Flats; and when they can, trading the Flats for larger Cubes. In this way they are getting used to the idea of the equivalence of a Rod and x number of Tiny Cubes, etc. This equivalent exchange is the basis for understanding the place value system, and at this point, we use base 3 only because 3 is an amount that nearly any child of 5 or 6 can manage. Otherwise, the algebra for the system is the same for any base.

① The blocks are, or show, numeric equivalence by congruence. There exist Tiny Cubes as units, Rods equivalent to "x" number of Tiny Cubes, and Flats, equivalent to "x" number of Rods or "x²" number of Tiny Cubes. (Etc. to largest block in the set.)

② The game of super block is played with: 4 or 5 people, a sorted pile of blocks--in whatever base you have, regular dice, and control cards, one card per person.

③ Game rules are: Each person rolls the die, or dice to decide a beginner--high or low number goes first. Once the beginner or starter is determined, play continues clockwise.

④ Object of the game is, by taking turns rolling the die, to be first to acquire the Super block (x^3 of x^3, depending on base).

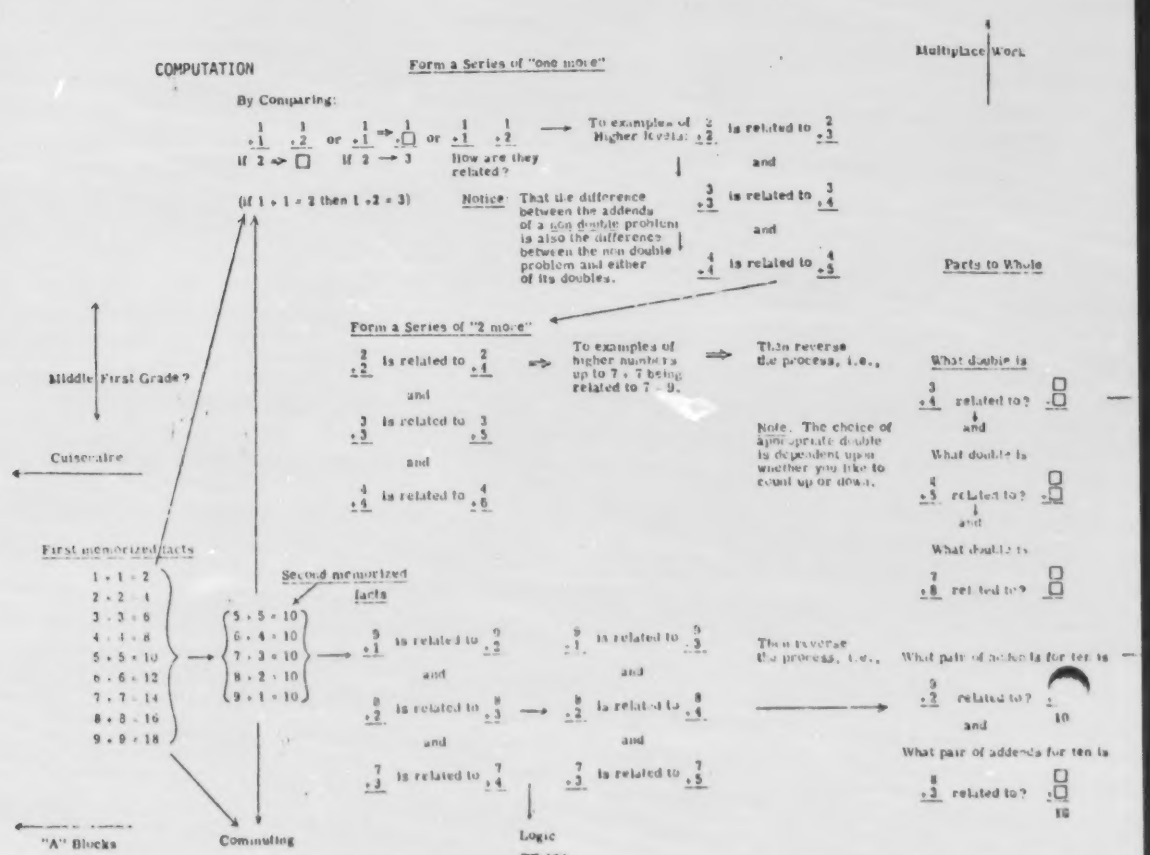
⑤ Method of play: Player rolls the die and acquires an amount of x^0 equal to dots on die face. By exchanging x^0 up to x^1 blocks and x^1 up to x^2 up to an x^3 block he accumulates and [illegible]s accordingly until someone is first to acquire one x^3 or a block.

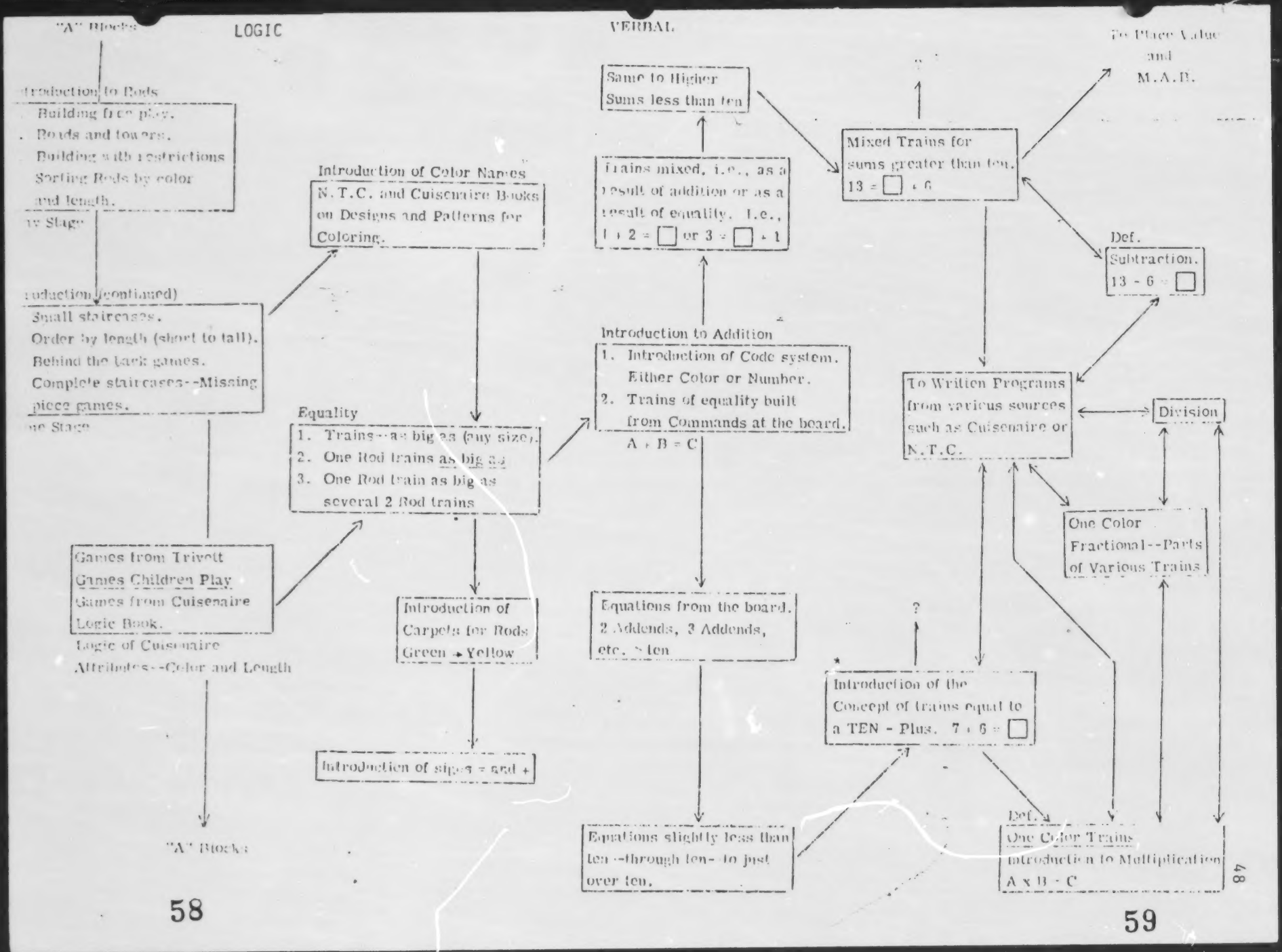
Sample Cards and Discussions (SB_{1-3})

Teacher control in these games is paramount to direct learning from these activities. What follows then is an account designed to help you control and otherwise direct the insights children will be confronted with because of the design of the game.

Suppose the game is base 3. Player one has rolled the die and it turns up ⁘. He may do the following ① He might count 5 Tiny Cubes, place them on his card and ignore his exchange. You must observe this and correct it for he must trade while he has the die. ② He might count them onto his card and then trade, thus having 1 Rod and 2 Cubes. At this point (the first few games), that's acceptable play. ③ He might reach into the pile and pull out 1 Rod and 2 Cubes indicating to you a complex understanding of the game already.

It is ③ above that we are striving toward, i.e., an understanding on an intuitive level that 5 is 1 Rod and 2 Cubes. It should be understood that the design of these activities precludes ③ and you probably won't see anyone do anything except follow your explicit directions. This is an extremely important pedagogical note to remember, in that all pre-planned activities provide the Teacher to some extent with this problem. If there is some deduction you wish the children to draw from the activity, the rules or planning itself may obviate the child's "seeing" the idea until such time as the rules for the game are internalized and out-of-the-way, then he might have time to deduce the obvious or not so obvious. Take your time, play these games until the children are really good at playing them. Emphasize the conversations that are wrapped around the activity, listen to their language and improve it. Keep in mind where you are going--at some point you want to have conversations with them concerning place value--tens and ones--carrying and borrowing will come out of this game, how it comes out depends on how clever and observant you are, and how well you train your children is being clever and observant.


COMPUTATION
Form a Series of "one more"
Multiplication Work
By Comparing:
If 2 ⇒ □
If 2 → 3
How are they related?
(if 1 + 1 = 2 then 1 + 2 = 3)
Notice: That the difference between the addends of a non double problem is also the difference between the non double problem and either of its doubles.
To examples of Higher levels:
is related to
and
Parts to Whole
Form a Series of "2 more"
To examples of higher numbers up to 7 + 7 being related to 7 + 9.
This reverse the process, i.e.,
Note. The choice of appropriate double is dependent upon whether you like to count up or down.
What double is related to?
What double is
What double is
What pair of addends for ten is
Middle First Grade?
Cuisenaire
First memorized facts
1 + 1 = 2
2 + 2 = 4
3 + 3 = 6
4 + 4 = 8
5 + 5 = 10
6 + 6 = 12
7 + 7 = 14
8 + 8 = 16
9 + 9 = 18
Second memorized facts
5 + 5 = 10
6 + 4 = 10
7 + 3 = 10
8 + 2 = 10
9 + 1 = 10
Then reverse the process, i.e.,
"A" Blocks
Commuting
Logic


"A" Blocks
LOGIC
VERBAL
To Place Value and M.A.B.
troduction to Rods
Building free play.
Rods and towers.
Building with restrictions
Sorting Rods by color and length.
w Stage
Same to Higher
Sums less than ten
Mixed Trains for sums greater than ten.
13 = ☐ + 6
Introduction of Color Names
N.T.C. and Cuisenaire Books on Designs and Patterns for Coloring.
Trains mixed, i.e., as a result of addition or as a result of equality. I.e., 1 + 2 = ☐ or 3 = ☐ + 1
Def.
Subtraction.
13 - 6 = ☐
roduction (continued)
Small staircases.
Order by length (short to tall).
Behind the back games.
Complete staircases--Missing piece games.
ne Stage
Introduction to Addition
1. Introduction of Code system. Either Color or Number.
2. Trains of equality built from Commands at the board.
A + B = C
To Written Programs from various sources such as Cuisenaire or N.T.C.
Division
Equality
1. Trains--as big as (any size).
2. One Rod trains as big as
3. One Rod train as big as several 2 Rod trains
Games from Trivett Games Children Play
Games from Cuisenaire Logic Book.
Logic of Cuisenaire Attributes--Color and Length
One Color Fractional--Parts of Various Trains
Introduction of Carpets for Rods Green → Yellow
Equations from the board. 2 Addends, 3 Addends, etc. > ten
?
Introduction of the Concept of trains equal to a TEN - Plus. 7 + 6 = ☐
Introduction of signs = and +
"A" Blocks
Equations slightly less than ten --through ten-- to just over ten.
Def.
One Color Trains
Introduction to Multiplication
A x B = C
48
58
59

Lists of Suitable Materials for
Primary School Mathematics

Prepared by Harold Taylor

Creative Publications
3977 East Bayshore Road
P. O. Box 10328
Palo Alto, California 94303

		Price
I.	Operations on Whole Numbers	
A.	Cuisenaire Rods	
1.	Classroom Set (wood) 32 paks and Instruction Manual	168.95
2.	8 paks without guide	40.00
3.	Idea Book for Cuisenaire Rods at the Primary Level	9.95
4.	One Digit Addition and Subtraction With Cuisenaire Rods Teachers Resource Guide	9.50
5.	One Digit Addition and Subtraction With Cuisenaire Rods Teachers Set (Includes Teachers Resource Guide and Student Work Book)	11.95
6.	Cuisenaire Rod for the Overhead Projector	4.95
7.	Using the Cuisenaire Rods - A Photo/Text Guide for Teachers	8.95
B.	Unifix	
1.	Unifix Elementary Kit (500 unifix cubes; two each of 1-10 stairs, number indicators set, 10 x 10 insert pattern boards, etc., teachers manual.)	88.00
2.	Group Work With Unifix Structural Material (Manual)	.25
3.	Unifix Teachers Manual	1.00
4.	Unifix Multibase Structures Guide	.25
5.	Unimath (Scaffa & Castellono). Over 50 classroom tested activities dealing with pre-number and number concepts, numeration, operations with whole numbers, estimation, measurement, etc.	7.95
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1.	The Math Balance. Very sturdy 10 1/2" high balance which allows the child to deal with abstract mathematical ideas by manipulating a physical model.	13.50
2.	Math Balance Activity Cards	4.50
3.	Math Balance Teachers Guide	2.25
D.	Patterns to Play on a Hundred Chart. Generate more than 44 visual patterns on a hundred-board, a good way to practice computation and problem solving.	5.75

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3.	Unifix Teachers Manual	1.00
4.	Unifix Multibase Structures Guide	.25
5.	Unimath (Scaffa & Castellono). Over 50 classroom tested activities dealing with pre-number and number concepts, numeration, operations with whole numbers, estimation, measurement, etc.	7.95
C.	The Math Balance	
1.	The Math Balance. Very sturdy 10 1/2" high balance which allows the child to deal with abstract mathematical ideas by manipulating a physical model.	13.50
2.	Math Balance Activity Cards	4.50
3.	Math Balance Teachers Guide	2.25
D.	Patterns to Play on a Hundred Chart. Generate more than 44 visual patterns on a hundred-board, a good way to practice computation and problem solving.	5.75

II. Place Value

A. Number Blox Math Labset. 1200 units, 240 tens, 80 hundreds, 10 thousands, and Activities Books A and B. 195.00

B. Number Blox Activity Book A. Activities K-6. 4.95

C. Number Blox Activity Book B. Activities 2-6. 4.95

D. Picturing Arithmetic Series. 4 books. 25.00
   1. Numeration 7.50
   2. Addition 7.50
   3. Subtraction 7.50
   4. Multiplication and Division 6.25

   Excellent sources for use with Number (Base 10) Blox.

E. Multibase Arithmetic Block, Set. 180.00
   Contains bases 2,3,4,5, and 10 (classroom set)

F. Multibase Activities
   1. Base 4 4.25
   2. Base 5 4.25
   3. Base 10 5.00

G. Chip Trading. Math Lab Set. Enough materials - chips, control cards, abacii, etc., for 12 students with manuals. 108.00

III. Fractions

A. Cuisenaire Rods. See Addition and Subtraction of Whole Numbers.

B. Fraction Bars. Junior High Set. Enough materials for 30 students with manuals, workbooks and activities. 143.00

C. Fractions With Pattern Blocks. Good ideas on using pattern blocks for teaching fractions in grades 2-6. 5.95

IV. Logical Thinking, Problem Solving

A. Attribute Materials:

   1. Attribute Games and Activities. 155 pages of games and activities to develop mathematical concepts. 6.00
   2. Loop Activity Cards. Activity cards to accompany book. 13.00
   3. Combined. 16.00
   4. Attributes Games and Problems Set. ESS materials developed at EDC, Boston. Materials for above books includes A-blocks, people pieces, color cubes. 15.00

B. Desk Top Attribute Blocks. (British Style) These blocks are different from ESS A-Blocks but allow for a unique change in attributes that can give greater flexibility than might be allowed using only one type of blocks. 13.50

C. Various Mastermind Games, ranging in price from 2.75 - 13.75

D. A wide variety of games for strategy and logical thinking.

1.	Backgammon	2.95
2.	Othello	11.50
3.	Battleship	9.50
4.	Pathfinder	10.95
5.	Chess and Checkers	4.50
6.	Dominoes - double six	2.95
	double nine	5.00
7.	Mancala - Ancient African game	12.50
8.	Tower Puzzles	6.25
9.	Fox and Geese	7.50
10.	Nine Men's Morris	7.50
11.	The History of Board Games. A book that traces the background of board games. Outlines rules and boards for 66 games.	5.00
12.	3D. Tic Tac Toe	9.25
13.	Somma Puzzle Solutions	1.95
14.	Somma Puzzle	4.50

E. Publications

1.	Finite Differences, Seymour and Shedd. Discusses systematic approach to problem solving.	5.75
2.	Thinking Through Problems, Barnabus Hughes. A manual of heuristics.	5.50
3.	The Book of Think, Marilyn Burns.	5.00
4.	Successful Problem Solving Techniques, Greenes, Gregory and Seymour. Outlines and describes 9 basic techniques for solving problems.	5.95

Cuisenaire Company of America, Inc.
12 Church Street
New Rochelle, NY 10805

I. Operations on Whole Numbers and Fractions

A. The Rods

1.	Kindergarten Set (In wood)	84.00
2.	First Grade Set	98.00
3.	Second Grade Set	101.00
4.	Intermediate Set (3-8)	83.00

Each kit contains Rods and activities especially designed for the specific grades indicated.

5.	Idea Book for Cuisenaire Rods (K-2)	9.95
6.	One Digit Addition and Subtraction (1-2)	
	Pupil and Teachers Set	11.95
	Teacher Guide only	9.50
	Workbook only	2.95
	Set of 10 workbooks	27.50
7.	Picture Puzzles with Cuisenaire Rods (1-6)	3.95
8.	Multiplication Division with Cuisenaire Rods and Graph Paper (3-8)	4.50
9.	Using Cuisenaire Rods - Jessica Davidson How To Book on Cuisenaire Rods Resources (1-8) for Teachers	8.95
10.	Cuisenaire Rods for Overhead Projector (K-8)	5.50

B. Place Value

1.	Starter Set for Powers of Ten (K-)	27.50
2.	Primary Powers of Ten (1-3)	56.00
3.	Intermediate Powers of Ten, with lines (4-8)	87.00
	Intermediate Powers of Ten, without lines (4-8)	82.00
4.	Powers of Ten for Overhead Projector	9.95
5.	Computing Worksheets and Duplicating Masters for base ten work	4.75
6.	Base Ten Mathematics, Mary Laycock 50 Activities Grades 2-8 Base Ten for use with Powers of Ten Materials (2-8)	4.95

C. Fractions

The 1980 Cuisenaire catalogue does not show any specific activities for fractions. However, the Jessica Davidson Manual Using the Cuisenaire Rods does have a section on fractions. However, let me say that Cuisenaire Rods are probably the best model for fractions instruction you can find and we are working on ideas that can be used with Cuisenaire Rods.

D. Logical Thinking and Problem Solving.

1.	Hidden Rods Hidden Numbers, Charbonneau 1-8.	2.95
2.	Games Children Play, Triveft K-6. One of the very best sources of games for young children.	7.00
3.	Krypto. Card games using 4 basic operations to solve problems, 3-8.	4.00
4.	Puzzle Tables. Problem solving and number drill in unique format, 3-8.	4.50
5.	2 4 6 8 Let's Start to Calculate, Jessica Davidson. Excellent source of ideas for calculator work that emphasizes how to learn math.	4.50
6.	Calculator Explorations and Problems, Miller. Another calculator exploration book that emphasizes computational algorithms, equations and problem solving.	4.95
7.	Wolly Goggles and Other Creatures. A book on guess my rule, and pattern solutions in problem solving.	4.50
8.	Move In On Math, D. Whittaker. High powered approach to arithmetic from problem solving format, 1-6.	7.95
9.	Attribute Logic Blocks (British Style) K-6.	15.00
10.	Attribute Blocks, D. A. Botterill, K-6. Manual of suggestions for developing programs in A. Blocks.	3.95
11.	People Piece Primer, McClean and Sternberg. Manual on ideas for using day to day experiences in logic and reasoning, K-3.	4.95
12.	Foo. Card game for the rules for the order of operations, 3-9.	6.95
13.	Domodots. Game emphasizing dot patterns as means to number concepts, K-3.	5.95
14.	Quick Master Chess. A new method for learning chess. Comes with 32 page book explaining rules of chess, K-12.	9.95
15.	A HA, Gardner. Book looks at the sudden hunch, the creative leap of the mind that "sees" in a flash how to solve a problem in a simple way, 7-12.	6.50

Society for Visual Education (S.V.E.)
1345 Diversey Parkway
Chicago, IL 60614

I. Basic Operations Emphasizing Place Value. Automatic advance or manual advance filmstrips on basic number concepts as well as complex operations. Prices range from $11.00 per filmstrip to as much as $365.00 for complete sets of filmstrips, tapes

and manipulatives. These are Lola May materials and the 1 to 10 and 10's to 1 exchanges are very clever. These must be viewed to be appreciated, they are first-class.

II. Problem Solving. Logical Thinking. There are four sets of problem solving materials that can be used in grades 3-9. These activities and filmstrips (manual or auto) emphasize approaches to solving several types of problems. Set B (63.00) and Set D (78.00) are particularly intriguing. Again these must be seen to be appreciated.

Midwest Publications
P. O. Box 448
Pacific Grove, CA 93950

I. Logical Thinking

A. Mind Benders, Harnadek. 8 books on deductive thinking skills. each 3.50
Or in duplicating masters each 9.95

B. Basic Thinking Skills, Harnadek. 11 books on skills such as Analogies, Antonyms, and Synonyms each 3.50
Or in duplicating masters each 9.95

C. Inductive Thinking Skills, Harnadek. 11 books on skills, such as Cause, Effect, Figure Patterns, Figure Relations A and B etc. each 3.50
Or in duplicating masters each 9.95

Midwest Publications has the best supply of this type of material. It emphasizes thinking in logical ways by Induction or Deduction. The problems have been presented to children in grades as low as 2nd and 3rd. This material is highly recommended.

LaPine (Mathematics) Scientific Corporation (several addresses)

Illinois - Dept. C4321, 6007 South Knox Avenue, Chicago, IL 60629
New Jersey - Dept. C4321 377 Chestnut Street, Norwood, NJ 07648
California - Dept. C4321, 980 Parker Street, Berkeley, CA 94710

The LaPine Fractions material is broader than found elsewhere, and much of their material is high quality, hands on models.

Within their catalogue you will find chip trading, multibase blocks, cuisenaire and the like along with magnetic number lines (chalk boards for study of integers) large (12 element) chalk holders for making graphs on chalk boards; special tools for the math lab such as clinometers, theodolites, tape measures, timers, and measuring tools of every sort. This is a catalogue every elementary teacher should have.

Using Cuisenaire Rods (Two Cases)

--"It wasn't perfect."

Bernadine Stake

"If you asked about reading, you'd be there all day," said one elementary school principal. The question then, is why don't teachers discuss the teaching of math as much as they do reading? For many people and especially for women, mathematics is a difficult subject. Uneasiness about this difficulty may account for their reluctance to discuss the subject.

We learned much from the teachers and the children as we worked with them on problems identified by teachers. We found the teachers knowledgeable about children and children's abilities in mathematics. They are also inventive and find creative ways to work with children on problems in mathematics. However they do seem reluctant to share those ideas with others.

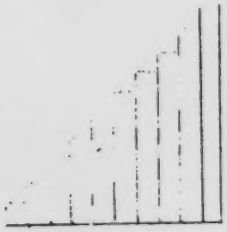
We learned that talking about mathematics was easier when the resource person and the classroom teacher worked as a team on a common problem. One example of a cooperative effort was in response to Ms. Judson's expressed difficulty in teaching borrowing and carrying.

I suggested that we use the Cuisenaire rods to help the children understand the principles of adding and subtracting the ones, tens, and hundreds. As a team, Ms. Judson and I introduced the Cuisenaire rods to her second grade class. The children built structures with the rods to become familiar with these materials before we introduced the concepts of borrowing and carrying.

The following is a transcript of Ms. Judson teaching a lesson on borrowing and carrying, which began as my review ended. This record contains most of the teacher's conversation. In most instances, the children's responses

were too faint to record. One thing that struck me was the repetition, indicated by the letter "R" in the left margin.

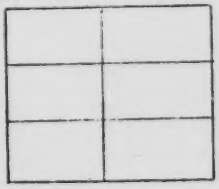
Ms. J. Okay, remember when I gave you that piece of white paper and you did some renaming or some borrowing. We're going to need some table space again. Don't break up your stair steps.



(These "staircases" had been created from Cuisenaire rods during the review.)

Ms. J. Slide the others (rods) back into the junk pile. Leave your stair steps so you can see them. Pam, you weren't here were you? These are the same papers we used yesterday. Okay, please put your white paper in the middle of your desk. All right, let's try some problems.

* Two sets of ten.



(Children find 2 orange rods.)

R See if you remember. Two sets of ten. Now that's the second time I've said it. Remember the first thing you did yesterday. Do you?

R Two sets of ten. Let's try three ones in the one's box. If you don't have three ones go to the supply box. Jerrard, do you have three ones? Webster, do you know the name of this picture in real numbers? What would the number be that stands for this picture.

(Webster thinks.)

Ms. J. Give him a minute to think. I'm hearing some buzzing over here. That
R doesn't help, Ann. Ann, how many sets of ten do you have?

Ann: Two.

R Ms. J. Okay, two goes in the ten's column. Two sets of ten stands for 20, doesn't it?

R Two sets of ten. How many ones do we have, Webster? Now in ordinary talk, what do we really name that number?

Webster: 23

Ms. J. Okay, Now we make a minus, a subtraction problem out of it. I * would like you to please put five little cubes in the one's column. We're subtracting. If you don't have five little bitty ones, raise your hand and we'll give you some more.

(Ms. Judson looks at a child's paper and sees that he has put the five ones with the three ones.)

Ms. J. Whoops, they have to be downstairs. We're taking away, downstairs, Betty.
R Five cubes in the one's column down below. Five down below. They
R have to be in the one's side. Why is it important to have these
R ones over here in this box? (one's side) How come I couldn't just put these ones over here? (on tens side) Why does it make any difference at all? Susan?

Why is this redundancy necessary? I noticed that when teaching a whole group new concepts, Ms. Judson was very careful to see that each child was following the lesson. The continuation of the lesson transcribed below shows just what a struggle it was to get the children to follow the idea of representing tens and ones with the rods.

Ms. J. You know what, Mike, maybe I should have done this, written the word "ones" up above here and "tens" on this side. Maybe that would have helped you. Now, the question of the hour: Delbert, you have three ones, and you're supposed to take five away from the set of three.

Delbert: You can't do it.

Ms. J. Say it again. Say it again. Oh my, what a tragedy. What do we do?

(no response)

Oh, we just forget the problem, right?

(no response)

Oh, Delbert, what shall we do?

(no response)

All right. Ted, would you please tell us where in the world, what in the world do we need?

Ted: Knock him out?

(holds one of the tens)

Ms. J. He's moving next door, isn't he?
He's just moving next door, isn't he? He went next door and said, can I move in over here? He's the very same problem. He just moved next door. So now ... Check it first. How many tens do we have now?

Amy: one

Ms. J. How many ones do we now have?

Webster: thirteen

Ms. J. You got it. It's 13. Listen, I'm hearing a lot of buzzing. Delbert, is 13 large enough to take five ones away from it?

Delbert: Yes, uh huh.

Ms. J. All right. Let's try it. Let's make a long train with your orange bar and your three ones.
All right. I'm going to show you how to do it for those of you who don't remember yesterday. Line up your 13 ones. Charlotte's doing fine. Ben, you with us?
Jacob, I don't see your 13 ones lined up like a train. Would you please line them up. Take your ten bar, add your three ones on. Make a 13. Take your five ones that you're subtracting. Lay them on top. Those of you that have a 13 train, lay your five ones on it. No, Jacob, we're making a train of 13 ones. Lay your five ones along the side. This is what you're taking away. Now, wait a second, Nancy. Are you holding on to those five ones, or are you taking them away?
Well, whoops. Show me your 13 train. Show me a train of 13. To make it achieve a length of 13, you have to lay them all on the end. 10, 11, 12, 13. You're taking away, how many?
What bar would make up the difference?
....
You need to find a bar. If you know it, raise your hand. Ben, work it out. You got it. Raise your hand. Lucy, I want you to show me. Jacob, you got it?
Raise your hand and let me know. What's going to fit in there?
Find a bar that will fit in there, Jacob.

The children were subtracting 5 from 23. The strategy Ms. Judson taught was to find the difference between thirteen and five. Some children were confused. They seemed to be thinking more about the procedures they were following than about the difference between 13 and 5, or about how much would be left if 5 were taken away from 23.

Ms. J. Mimi, you've got it? Mimi put the 8 rod along side the five ones to make thirteen using 8 and 5 ones. Is that right?

Dan: Yes

One, two, did Dan figure it out?

Children: Yes

Ms. J. Mimi needed an eight bar to match up with her five ones which would be the same length as the 13 she started with. But the question is what do we do with that one 10 that's left up in the 10's column, Daniel? (no answer)
Do we leave it up here in the ten's column?
It's not to be here by our answer, mimi. You can think of a zero sitting there holding that place open if you want to. There is nothing there. Just a place holder. Daniel, one set of ten, you take nothing away from it. What would we have left?

(To all children) Show me one set of ten. Pick it up for me. If you take nothing away from it how many sets of ten will you still have left?

Well, Eric, can you help us?
(no answer)

How many?
(no answer)

You have no idea what we're talking about, do you? (no answer)

Daniel, hold it up real high. Take nothing away from it. How many sets of ten do you still have left?

Daniel: One

One set of ten.

Ms. J. It's still there. You still have one set of ten. This table is about to put away their pieces. Don't compete with your voices. If you have a question, we'll be glad to help you. Daniel, how would you read your answer?

Daniel: 18

Ms. J. Boys and girls, I think we're going to stop here today. Maybe tomorrow we'll have some more time.

Obviously teaching subtraction was not an easy task. After the lesson Ms. Judson and I discussed subtraction and, in so doing, created a new system which we thought would be less confusing to the children.

Using the same problem:

tens	ones
2	3
-	5

We decided to put the two tens and the three ones in the top boxes respectively and then give attention to the meaning of subtraction. Since five cubes were to be taken away from the twenty three cubes, we decided that no Cuisenaire rods would be placed in the space after the subtraction sign. We also planned to discuss with the children why no ten rods could be placed in the ones column and no unit cubes could be placed in the tens place. We thought this might be more meaningful to the children and easier as a way to introduce subtracting with the Cuisenaire rods in the second grade than finding the difference.

Together we created this system to help the children understand the concepts of borrowing and carrying. The team work was important to the generation of new ideas. Ms. Judson said, "Sure think it would be a better way." She also said that she was impressed with the Cuisenaire rods and, if she had seen how valuable they were in the fall, she would have used them all year. She said, "I really learned a lot from them. I want a set for my room so I don't have to borrow them each time I want to use them for math next year. The children really began to see the rods as math materials and not just building blocks although they enjoyed them as building blocks also."

Ms. Judson had been reluctant to use Cuisenaire rods at the beginning of the year, and they had evidently not seemed attractive when a set we left at the school the year before. One day in April, I asked her if Hal Taylor could demonstrate the use of the rods in her room. She reluctantly agreed, saying that her children needed to become used to other adults as they gave substitutes a hard time.

The subsequent week I worked with the children on fractions. I had difficulty gauging the children's level of sophistication, so the lesson could have been better. Ms. Judson and Dr. Easley came to my rescue and the lesson

continued. Team work was much appreciated in this instance. When researchers try to do what teachers do they make themselves just as vulnerable as the teachers are when we observe them. Somehow I think this builds up trust between the teacher and the researcher. At any rate, Ms. Judson seemed quite happy to have my help from then on, and she began to try new things with my help. Perhaps the two instances of the so called "experts"; having difficulties gave Ms. Judson renewed confidence in her ability as a teacher.

It is important to have some degree of harmony in the classroom. Whether interactions in the classroom are harmonious or not seems to depend upon both children and the teacher. As resource people, we tried not to disrupt and cause chaos in the classroom. We found that children have certain expectations of what goes on in their classroom and want the visitors to observe the rules of their culture. For example, one child told me, "When the kids are too noisy, say, 'one, two' and then they will stop. Starting to count had meaning to the children while my more general question, 'Could you please be quiet and listen?' did not have the same effect. Children were aware of what I wanted, but somehow they did not seem to be capable of responding. They like the rituals that have been established and respond to them.

Learning to be a part of the culture of the school is important for feeling accepted which is necessary if one is going to help the teachers and children try something new. For without an accepted basis of behavior, nothing new can be built. It's not simple like trying a new dish on the family at dinner time. Cuisenaire rods could change the whole ritual of math lessons, like introducing Middle Eastern folk dancing into an Episcopal service, it could upset a lot of people who expected something else.

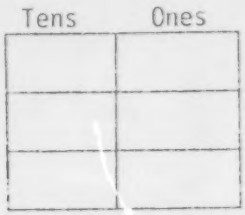
Another aspect of my work as a resource person was relieving a teacher of her responsibilities for a particular group so she could concentrate on teaching the others. Some teachers identified children who were having

difficulties and needed tutoring. One teacher identified a group of gifted children who needed help on borrowing and carrying (a very common problem in the primary grades). To teach borrowing and carrying with this group I used the new strategy that Ms. Judson and I had developed for her class. A transcript of part of the lesson follows, to which I have added a few interpretive comments.

5/21/80

Five children (C) and myself (Adult A) are sitting around a small table.

A: Okay, now I want you to clear enough space that you can put this white sheet of paper in front of you. (Children push their Cuisenaire rods to the side and place the papers in front of them.) That's good, you'll need the rods to work some problems. (hands out sheets of paper)



C: Hey look, little picture frames.

A: Okay, push the rods together, that's lovely. You've made some nice things. (Children have built structures with the rods and now are getting ready to work other problems.) Now we are going to do some renaming. Do you know how to rename?

C: No.

A: Okay, so we'll do some problems with the rods. First, we're going to add. I want you to look at the top of this paper. It says tens and ones. Never, never can you have ones in the ten's place and never, never can you have tens or other rods in the one's place.

C: One ten and a one doesn't fit like this. (Child had placed both in the ten's column.)

A: No. Let's put 11 on the top. Don't write it. I don't want you to write it. I just want you to use the Cuisenaire rods. Eleven. Where do you put the tens?

C: Here. (Child places one ten correctly) Tens, ones.

A: Put the ten on the tens side. Whoops! (to one child) You got them turned around. Tens and ones. Okay, now, how do we read that number? (All children had now placed the rods correctly.)

C: Eleven
C: Eleven.

A: Eleven, isn't it?

I was trying to help the children understand the meaning of the decimal system. The children seemed to be following my directions,

so I assumed that they understood. Children were talking to each other as well as to me; and it was not until I listened to the tape that I realized I had not responded to the next remark, or even noticed it.

C: One and one is eleven.

The child may have understood, but couldn't verbalize her thinking, or she could have been confused. Who knows whether she got straightened out or not?

A: All right, now we're going to add that to another number. We're going to add, so we're going to put 55 under the 11. No, don't write it. Put 55 in the next box down below. - 55.

C: This is take away.

A: Oh, I don't want you to take away. We're going to add first. We may take away next time. Fifty five. Now remember here we can only have ones. (Points to paper under one unit cube.) So use ones. Only ones here. Make 55.

I should have been asking more questions of the children. I was surprised to find that I had told the children what to do so much of the time, rather than asking them what they forget?

C: O yeah, you have to have ones.

A: Right, just use ones.

C: I need one more. (Children are counting out the five ones.)

C: One more.

C: Three, four, five.

A: So you have 11 and 55. Now let's add that. You bring them down here and find out how much it is. Bring all of these units and tens down to the bottom.

C: Sixty six.

C: You're not supposed to write it?

A: No, you're not supposed to write, because we're going to use this paper over and over. So, what do you have? You bring them all down and what do you have?

C: Sixty six.

They seemed to catch on very easily to this simple addition problem, so I went on to a problem that involved carrying.

C: Sixty-six.

A: Very good. Okay, let's try another one. This one, I want you to take 49, 49 plus 35.

C: I got 'em.

A: Okay, tell us what your answer is.

C: Fifty, no 80.

A: You just think it to yourself.

C: Eighty-four.

A: Let everybody else figure it out, too.

Children: Lots of noise and talking in the classroom (not understandable)

C: Thirty-five.

C: Is it 35?

A: It's not 35.

At this point the children seemed to be trying very hard to figure out the answer to the problem. They talked to each other, to me, and to themselves. The dialogue continues sloppy now. All but one child traded in ones for a ten and carried the ten to the ten's place. With Sara's help Betty seemed to understand as well.

C: Forty-nine and 35. Mrs. Stake, is it 49?

A: What?

C: Or 83.

A: Just a minute and you can tell them how you did it. I like the way you did it. Just wait a minute.

C: There, I got my answer.

A: Where's your answer?

C: I'm going to trade this in for a ten.

A: Forty-nine. First you had your 49.

C: I need one ten.

C: I trade that ten.

C: It's 84.

A: Okay. so you're carrying. I want Sara to show you how, tell them. You listen, Betty, because you didn't quite get it yet. Okay, you show them how you did the whole thing. Start from the beginning.

C: Yeah, 70.

A: Yeah, 35 and 49, right?

A: Let's see. How many are there?

C: Nine.

A: All right. Now, Betty, get yours, 49.

C: Oh, Sara

A: And then Sara's going to show you how to do it. All right Sara. See you all got it. So you could figure out another problem. You could go on. You see, she brings all those down. How many is that all together?

C: Forty-nine, 49, 35 (mumbling)

A: See, she brings all those down and how many is that all together.

C: Fourteen.

A: Fourteen. Can you have 14 there? (points to ones)

C: No

A: So what do you do?

C: You have to carry.

A: Carry one what?

C: Carry one ten.

A: So you have to carry one ten. You have to trade these in for a ten. Trade ten of them in. Trade them in. Put them in here and get a ten instead. Cause you can't carry those over. You have to carry a ten, don't you?

A: Now what's your answer? Eighty-four. Okay. Very good. Now let's see if we can do another one.

The children worked several more addition problems involving carrying and then I introduced borrowing. These second graders had worked problems on paper involving borrowing and carrying, but they had not used the Cuisenaire rods to work problems before. They seemed to be catching on very fast. As I analyze the tape, now I wonder if they were following procedures more than understanding the concepts. I didn't ask enough questions to really know. They worked so well with the rods that I assumed they were understanding. At any rate, we went on to subtraction with borrowing. I used the method that Ms. Judson and I developed to teach the subtraction.

A: Okay, I want to do a subtraction problem this time. Let's say- 55, 55.

C: Plus.

A: Okay, I want you to do this one, Candy. Fifty-five, 55. Make 55. Okay, now if you had 55, there you have 55, don't you?

C: Um huh.

A: Now, I want you to take away from that. We're going to take away from it. So would you put anything down here?

C: Um huh.

A: No.

C: We're not going to?

A: No, if we're going to take it away, we don't have more do we?

C: No.

A: Put 55. Just leave your 55. Now I want you to take away from that; 55 take away six. Just take six away, but don't put six down there, don't put six down because you're taking away this time. Don't put any others down there. Fifty-five take away six. How do you do it?

C: Take away six.

C: Can't do it.

C: Yes you can.

A: How do you do it?

C: You borrow from the five. (child takes one ten from the five tens and trades it for ones.)

A: All right. So what do you do with this one? You can't put tens in there so you put one's in there, don't you?

C: Ten ones.

C: Three, four, five. I got it.

A: So.

C: One, two, three, four, five, six, seven, eight, nine, ten. Fourteen, take away.

A: Now you have ___

C: Nine.

A: That was 55 take away six, right.

C: Number six.

A: Six, now can you take six away when you borrow ones and make that fourteen.

C: I got it.

C: Forty-four.

C: No.

A: So what's your answer?

C: Fifty.

C: Forty-nine.

A: Very good. That's right. Did everybody get 49?

C: One, two, three, four

C: Yeah, I got it.

A: Okay.

C: Yes.

C: Yeah.

A: See we didn't put any in here. We didn't put any little ones in here. We didn't put any little ones in here or anything when we said take away because when you're taking something away, you don't add more to it, do you?

A: Do you see what I mean? Okay, now let's try this one. Let's try 37.

Some of the answers of the children are unintelligible on the tape. The method seemed to be working for bright children. It would have been better to use story problems. The children needed to be problem solving and the lesson was mostly following a procedure.

C: Thirty-seven. (Children worked more subtraction problems)

During the subtraction exercises some children asked to make patterns. They had made patterns of equivalence with the Cuisenaire rods on another day and wanted to do so again. The following is part of the conversation the child and I had as they made patterns and talked about the patterns.

I noted that one child was finding equivalencies of twenty by fitting as many of the twos, threes, fours, fives, etc. as she could beside the train of twenty. The patterns that the children created provided good opportunities to teach number concepts. Children were surprised to find that their patterns were representative of multiplication.

A: Do you know what you're doing? You're doing multiplication.

C: She is? Using multiplication?

The children were surprised to find that they were multiplying.

A: She's figuring out how many sets of two are in 20, how many sets of three.

C: I am too.

A: And how many sets of four.

C: And how many sets of five, and six.

A: See with the threes, there are some left over, aren't there? It doesn't come out exactly even for three, but it does for twos. It does for fours.

C: That's not what I'm doing.

A: Fives.

C: It does for ones.

A: Um huh.

C: Look at me. I'm making a wall.

A: And Harold and Sara are making lovely patterns.

C: This is nice.

A: Yes.

C: Doing it double.

A: Um huh. Um huh.

C: Two.

C: This is like you're outside when you're building and you're building the tallest skyscraper in the world.

A: Okay. You tell us, what makes 20?

C: What makes 20?

A: um huh.

C: Two nines, two nines, one two,

A: Make twenty.

C: Fours, five threes, five fours.

A: Five fours. Are those fours? What are they?

C: Fives. Four fives.

A: Four fives make 20.

C: Two eights and one four make 20. Sevens and six. Three six and two ones.

A: Make 20.

C: Make ten twos.

A: Um huh.

C: Six threes, one two make 20 and 20 ones is 20.

A: Twenty ones.

C: And six ones, three ones, four ones, five ones and four are ___ 20.

A: Wait a minute. Wait a minute. On that one, let's see, this is six, one six,

Children became confused by six ones or 1 six.

C: One six.

A: One three

C: One four, one five, one two.

A: And one two make what?

C: Twenty.

A; One five.

C: One five, one nine, one six make twenty, one three, one one, two.

A; One two.

C: One two. One four. One one, two.

A: One two.

C: One three, one four, one six, two fives.

A: Make what?

C: Twenty.

A: Twenty. Who else is ready?

C: I'm done.

C: I need some fours. I need lots of fours.

A: Okay. Harold, why don't you go agead.

C: All right. I got.

C: I can't find ones.

A: I'll find them, just be quiet.

C: Two tens make 20.

C: I would use eight if I were you.

C: One ones, one ones, and one nine, and two nines. No, two ones and two nines make 20. Two twos and two ones, two eights make 20. Two threes and two __.

A: Just be quiet for just a minute. And two sevens, right?

C: And two sevens, yeah. Two fours and two greens.

A: What are the green's?

C: Dark greens. Six make 20. Five fours make, no.

A: Four?

C: Four fives make 20. Six, two sixes and two fours make 20. Two sevens and two threes make 20. Two eights and two twos make 20. Two nines and two ones together make 20.

A: Very good. Plain old two tens, right? Real good, Harold. Okay. Okay, Jenny.

C: Okay, two tens equals 20. Two nines and one two equals 20. Two eights and two twos equals 20. Three sixes equals and one two equals 20. Four fives equals 20. Eight, one eight, one nine and one three equals 20. One seven, one three, one six.

A: Are you right here?

C: Um huh. One two, one three, one seven and one eight is twenty. one nine, one eight and one three equals 20. Ten ones and one ten equals 20.

When I listened to the tape recording of the lesson I realized that although I had felt the lesson went very well when I was working with the children, there were many ways to improve it. We should have discussed the meaning of tens and ones as the children borrowed and carried. The children should have been asked to think of ways to check their answers to find out if they are right, rather than depending so much upon the teacher.

It's not easy to teach math, even with Cuisenaire rods. They did make it easier for me to teach, but in some ways the rods made it more difficult for Ms. Jordan. She had a way of teaching which was comfortable for her. Asking her to change to a new method was somewhat similar to asking her to speak a new language. Moving from paper and pencil arithmetic to teaching math with manipulatives takes much effort and new learning.

The children, on the other hand, quickly adapted to the new materials. They enjoyed Cuisenaire rods. They often asked me in the hallways if I would come in their room so they could "do patterns." The children found the rods enjoyable, there was some assurance that they understood the mathematical concepts, but there was also the possibility that the children were picking up procedural cues and not basic conceptual understandings.

In my dissertation research (Stake, 1980), I interviewed primary children and found that too much attention to procedures for counting hindered the understanding of quantity. In this instance, my analysis of the tape showed that I too was teaching procedures too much. Keeping the children quiet becomes important, and it's easier to do so when the adult is dominating the situation. So, even though I had the Cuisenaire rods and was interested

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in teaching understanding, the control of the situation sometimes became my pre-occupation, and I kept the children quiet and directed rather than drawing out their concepts of numbers and base ten numerals. Teaching with the Cuisenaire rods is certainly not simple.

photo=Ivan Ruiz

The Problem of Mathematics Equity -- A Possible Social Mechanism *

Jack Easley

The following is, very much, a plan for further work, and a critique of some of the weaknesses in our present state of knowledge of relevant factors affecting the achievement of girls and minorities in mathematics. Since it reflects my own analysis of lack of equity and general quality in mathematics education in American schools (Easley, 1978, 1979, 1980), the premises of my argument should be made clear at the outset.

There are two principal ways in which students enter those advanced mathematics courses that signal they have a good prospect for success in a mathematical career. One is by dutifully memorizing (not necessarily by rote) the algorithms and definitions of each course in mathematics from kindergarten through high school and practicing their application on the set problems given in each course. The second way involves personally reconstructing mathematical ideas, trying them out on whatever problems one thinks of, generally staying ahead of the courses, reading math on one's own initiative and interpreting what is taught in the light of ideas which are often completely unavailable to the other members of the class. The schools are geared to produce the first type of mathematics student, and the second type of mathematics enthusiast schools are usually not prepared to support. Yet it is the latter, encouraged perhaps by persons outside the school, who tend to make the more impressive records in university level mathematics courses and careers.

Traditionally, in a segregated, male-dominated society, mathematics was a masculine subject, and both pathways were mainly taken by white, middle-class males and, to the extent that there was class mobility in mathematics,

* Based on a paper presented at a meeting to discuss the status of minority participation in mathematics. Ford Foundation, New York City, July 8, 1980.

the first of them may have been taken by a few white, lower-class males or even upper-class males. Today, with the schools committted to equality of educational opportunity, the first path is theoretically open to everyone. It is seen as requiring hard work, discipline, and stick-to-it-iveness, which are traditional goals of American public education. It is also the route advocated be Jesse Jackson and many leaders working for improved education for minorities. Teachers, parents, and children can work at this goal, understanding what they are about, for the curriculum is laid out in a linear fashion. However, it is not feasible to expect minorities or females in one generation to catch up with white males, either in representation or in levels of performance by this approach.

There are several reasons for this: First, persons who enter advanced mathematics by this route usually find themselves in a second class position in relation to some of the self-taught, creative problem-solvers, who readily invent their own ways of tackling new math problems because they always have. This level can be defined as the level at which it is impossible to make an "A" merely by diligent memorization of algorithms and developing skill in applying them to standard types of problems. It may vary from school to school, but it is the ideal to which most mathematicians who teach strive. Second, the independent attitude of the five percent or so of white, middle-class males who from kindergarten are taking the second route, may be the cause of considerable anxiety on the part of elementary school teachers who, generally being math anxious people, are easily perplexed by any show of creative mathematics. (One elementary school principal said to me that, "creative mathematics is a contradiction in terms.") Furthermore, as Morning, Mullins, and Penick point out, minority children may naturally develop a negative

attitude toward mathematics in reaction to the arrogance of these mathematically enthusiastic children. Third, elementary school teachers are not likely to tolerate creative mathematical problem solving from children for whom, because they are female, lower-class, or members of a minority, there is a special, official commitment--unless the teacher understands clearly that the work they are doing is of high quality. Being typically a math anxious person and committed to do her best for them, such a teacher tends to believe that all but those unusually talented in math have to follow standard procedures in order to make it as far as they can in high school and/or college math by hard work, thus proving themselves at every point as deserving of further opportunity or not. Such a teacher tends to allow only middle-class white males who show mathematical independence to go their own way.

There is more to it than that, because sometimes these white male math enthusiasts are so openly arrogant that it disturbs the best presentations of what the teacher wants to do for the rest of the class. Ignoring these boys is, therefore, a reasonably safe way out of a difficult situation. Allowing them this freedom, however, only whets their appetite for doing things their own way. I was one of those boys who refused to learn addition and multiplication tables, but always laboriously figured out each problem as a challenge deserving full creative effort. The teacher must hope that somewhere, in algebra or geometry, such boys will learn a little discipline. Unlike girls, minorities and lower-class whites, it is safe to assume that these boys will make it somehow into a math class where the teacher understands what they are doing. Meanwhile, their deviations can be ignored, but the other children must be taught to buckle down and carefully follow the rules.

This is the "back to basics" strategy which Stake and Easley (1978) found came more from teachers than from parents or school boards. This source is understandable because of the teachers' responsibility to bring each child up to grade level.

The problem is to find a way in which elementary schools can give all children an experience of creative problem solving without abandoning the discipline that would be good for future mathematicians as well as non-mathematicians. This does not require training elementary school teachers in advanced mathematics. It may not even require having them do creative work in arithmetic. It does require careful analysis of the way teachers see their work in math teaching. Teachers probably instinctively know that letting a self-confident math enthusiastic child follow his own ideas is better for such children than submitting them to drill on standard algorithms which they refuse to follow. However, teachers may not know that not letting minorities and girls follow their own ideas results in lack of an equal opportunity to reach the highest levels of mathematics achievement. Teachers are not to be blamed, for they have not had a preparation that would permit them to cope with creative mathematical problem solving, either to enjoy it themselves or to assist children who are doing it. This, on my analysis, is the central problem that needs solving in order to achieve a viable equity in mathematics and mathematically based sciences: How can we help most elementary school teachers promote a useful blend of creative mathematical problem solving and disciplined work?

Brandau and Easley (1980) made a small exploratory probe into the problem of eliciting teacher interest in creative problem solving. We

noted that applying known algorithms to problems is usually thought of as problem solving by teachers in elementary schools but not so among math education specialists. So we called that "problem solving 1" and the creative form, "problem solving 2". We noted that teachers engage in problem solving 2 in the area of social problems in the classroom, in which they often come to question standard approaches and try to get to the bottom of a particularly difficult case by creative exploration, detective work, invention, and trial, but they seldom have such experiences in the area of mathematics. Teachers know the emotional states of frustration, frustration, frustration, and eventually elation and personal pride of accomplishment at a breakthrough. They may not associate these states with doing mathematics, or if they do, would take it as a sign of poor teaching that children had that much opportunity for frustration.

Learning to manage the emotional responses children have (sometimes the elation is a worse management problem than the frustration) is something in which most teachers have had no training and little experience. The two forms of problem solving seem to be so different and to require such different forms of teaching, that, without more experience in managing mathematical problem solving, we should not expect teachers to grasp how it is possible to do both in the same classroom. In addition, the fact that children of different cultures and sub-cultures may have different behavior patterns in regard to personal insight, e.g., whether to let it out or to keep it to oneself, creates another set of problems here. Furthermore, the effects of previous schooling in suppressing creativity in mathematics in some and leaving it un-disciplined in others, means that most elementary school

teachers from grade two on will have a difficult time bringing about an equal opportunity for participation.

Harold Taylor, the remedial mathematics teacher for the Lincoln Elementary School in Ypsilanti, Michigan, has developed ways of releasing creative problem solving in nearly all children and disciplining them in the etiquette of a problem solving classroom. Taylor has found a number of children who were sent to him for remedial work, whose test scores and classroom participation improved so they were well above average after only a year's experience in his math lab, in which they mainly did creative problem solving. He has also worked with most of the teachers in the school (approximately 60) showing them ways of encouraging problem solving of a creative type and of teaching the etiquette needed to make that possible. Some of the teachers are using these methods successfully. In the past year, Taylor has participated as a resource person in two schools I am working with in an attempt to find out what pedagogical problems, in mathematics, teachers want to solve and how they can be helped to solve them. We find some considerable success in this approach, utilizing Taylor's demonstrations of his methods and supporting teachers as they try to understand them and adapt them to their specific needs.

Teachers, in general, are inadequately prepared to understand the functioning of the processes of mathematical thinking in children. The incremental learning models they have been given traditionally and in many education courses do not suffice. For example, they are hard pressed to understand cognitive conflict between counting algorithms and place value algorithms, although they may recognize it, and they have great difficulty understanding the apparent forgetting of what has just been learned when

a new application is presented. They may also lack sufficient understanding of social interaction patterns in different ethnic groups. Lack of nodding in blacks, avoidance of eye contact in Puerto Ricans, and lack of competitiveness in Navaho have been cited as patterns which can cause white middle-class teachers to mis-judge a child's response. Unless the teacher's own knowledge system is taken into account, I would think no good is served by identification of lower expectation as a general factor.

In fact, one of the findings of Stake and Easley (1977) was that teachers are generally in agreement with the civil rights goals of the schools but feel that the added responsibility of equality of learning requires more resources than they have. For example, in schools of all kinds and situations, teachers identified the problem of helping the children who did not quickly learn the material presented as their major problem for which they did not have sufficient materials or other resources. They felt that the courts, legislatures, and other authorities had imposed these laudable goals on them as problems which they would have to solve somehow without adequate aid. Teachers in schools in which I have worked in the past two years were generally not appreciative of the general orientations they had been given to mainstreaming the handicapped because they did not thereby acquire any better understanding of how to cope with reading, math, or behavior problems, and the chief burden of mainstreaming would be to multiply these problems.

In short, most teachers are not skilled in analyzing the socio-psychological learning difficulties that occur in particular subject matters or in general behavior. They know how to present lessons in ways that are interesting and engaging to the majority of their pupils but worry a lot, often hopelessly,

about those who are not engaged. They are less willing to take on problems that their pupils may encounter in later years when they have to devote all their energies to the here and now to meet what they understand to be societal expectations of what should be presented in their grade.

This situation cannot but have an unfortunate effect on children who come from different cultures than the teacher, or for whatever reasons, exhibit different patterns of behavior than the majority. However, current research on problems experienced by teachers shows they can be helped if they set the agenda. It is my opinion, based on some years of work with teachers, that if we do not come to their aid with the kind of help they recognize as solving the problems they experience daily in the classroom, they will not succeed in solving the problems we both wish they could solve for society.

The evidence for the social mechanization I have presented lies in many different sources, a few of which are indicated below:

(1) Project SEED (Whalen and Hanley, 1979) arranges for elementary school teachers of disadvantaged children no longer to have the sole responsibility for teaching mathematics to their students. When they share it with someone who is not anxious about creative deviations from standard mathematical algorithms, impressive gains in learning mathematics occur. This supports the notion that the difficulty many able minority students have in learning mathematics, as compared with the small number of white male math enthusiasts lies in the lack of support they receive for their own creativity for teachers. The same may be true of many white male children who are not very confident about their mathematical ability.

(2) A series of studies of black children in African settings (Kamara, 1973 and Nyiti, 1973) and in two different settings in this country (Nicholson, in progress) showed that conservation of quantity, weight, and volume could be achieved by approximately the same percentage of children of a given age as are whites in European or American settings, provided sympathetic encouragement and, in the American case, supportive schemes were provided.

(3) The Case Studies in Science Education, directed by Stake and Easley (1978), presents a picture of extremely dull mathematics classes from kindergarten to grade twelve in all kinds of schools with all kinds of student populations. Furthermore, there was clear evidence that most teachers in the representative schools studied felt inadequate to help slow learners and felt that they were very poorly prepared to work with minority children in math and science areas. Indeed, they seemed to feel a special need to help minority students overcome the handicaps of background, but seemed to realize they were not succeeding although they were sincerely trying. Most teachers in the study were very critical of the courses they had had at universities and the in-service courses they were offered. No one was coming close to understanding the real problems they had to work out themselves or offering useful help in solving them.

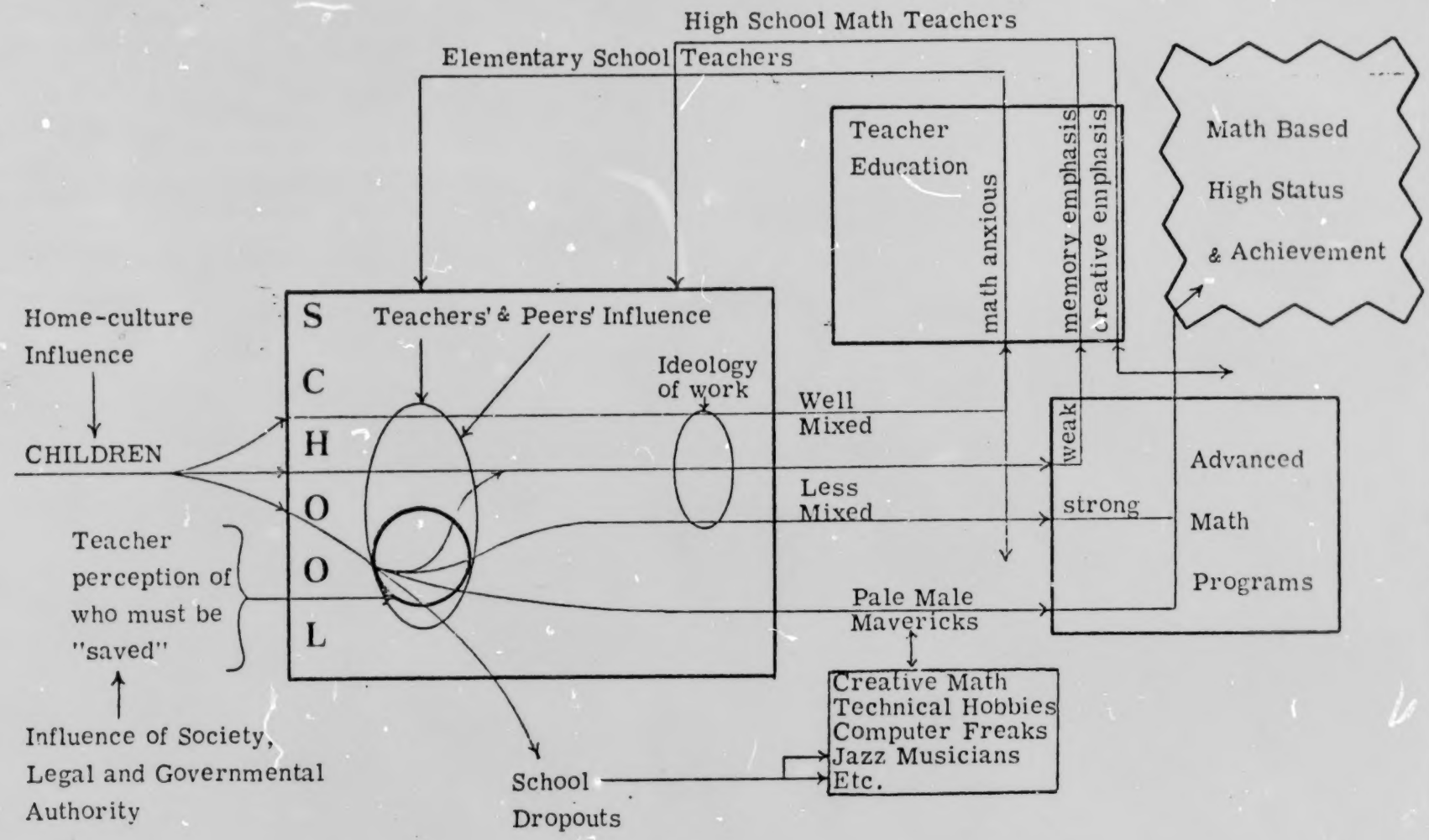
(4) The clinical interviews I directed in the NIE supported project on Children's Conceptions of Number and Numeral (Lerch, Easley, and Braunfeld, 1979) began with approximately 25 children each from a kindergarten and a second grade class in a school in which nearly half of the children were lower-class blacks and the other half were mostly middle-class whites. Sixteen children, eight from each class and approximately balanced racially, were selected to be followed for two years, with clincial interviews

approximately every two months about mathematical topics. Eight of these case studies were included in the final report to NIE. An excerpt from one of the eight, describing the responses of a black boy when he was in first grade, is appended to this paper. It illustrates the kinds of creative responses that would be very difficult for teachers to interpret without special training.

(5) Several sources of information make the strong suggestion that the basic algorithms supposed to provide meaningful learning of primary arithmetic, because they depend on the counting schema, described by Gelman and Gallistell (1978), limit the learning of facts of arithmetic and the development of mental agility in numerical thinking. First, a study of children's counting problems taken from the clinical interview data mentioned in (4) above (Stake, 1980), suggests that counting schemas can be extremely misleading and produce no understanding of quantity in themselves. Quantitative schema appear, when they do become involved, as if they come from a different part of the mind. Second, Wirtz (1980) finds that children use their intelligence more in primary arithmetic if they are not dependent on counting but can use different spatial patterns as representations of numbers. Finally, in my present research, studying teachers' concepts of mathematics and of math teaching in primary grades, several teachers have collaborated in trying out alternatives to counting, a problem which was suggested to me two years ago by one of the teachers I am working with, when she pointed out that there appeared to a serious conflict in children's minds between counting algorithms and place-value algorithms. It appears, now, that multiplication, which is involved in place-value algorithms, requires the flexibility of pattern thinking not possible for many children from

linear counting procedures. In short, the widespread dependance on counting algorithms (incidentally not found in Japanese primary math textbooks), although they are thought to provide a meaningful basis for arithmetic, do not allow most children the flexibility they need to think clearly about number operations other than adding and subtracting whole numbers less than ten. Pattern representations, furthermore, appear to provide a better basis for recall of these facts of addition.

(6) The IEA study of mathematics achievment in twelve countries (Husen, 1967) makes it clear that schooling in mathematics in the U.S. is distinctly not much above average for an economically developed country. As is now well known, the Japanese do far better in mathematics across the board. While there are numerous opinions about the explanation, their avoidance of dependence on counting algorithms may be a significant factor.



The critical decision point (circle) for minorities in mathematics and it's general ecology.

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Excerpt from a Case Study, "Art" by Jack Easley, from Lerch, et al (1979), "Children's Conceptions of Number and Numeral."

At one point the interviewer asked Art if he gave ten of his twenty-five chips to Mary, how many he would have left. He divided the chips into three groups, 10, 10 and 5, and replied that he would have ten left. This made little sense at the time, but reviewing the earlier interview, I found the problem to which this was the answer, namely, "Give ten of your chips to Shirley (he mis-remembered her name) and then give five to the interviewer."

Art's response to the 25-10 problem reinforces the notion, I first learned twenty years ago from David Page, that children's wrong answers are often answers to different questions. How is it that children understand our questions? If Piaget is right (see, also, Witz, 1973), children assimilate them to previous ideas they have been working with, whether they fit very well or not. That puts the burden on the interviewer to be alert to this possibility and to be prepared to listen to children's thinking very carefully. Any discrepancy from what would be a natural way of working should be taken as a signal that the child may be working on a different problem.

In December, Art was interviewed again, working on a 100 units long number line onto which Unifix cubes would fit. The chief point of the interview was to explore his quantitative feeling for the numbers from one to a hundred. One conceptual difficulty that was uncovered and clarified has to do with the tendency to see the number assigned the last item of a collection as the number of items in the collection even when the first number assigned to that collection is not one. For example, when he had two sticks of 20 orange unifix cubes each (he had selected them from the box, orange being his favorite color), he laid them end-to-end on the number line and read off how many he had

in each, "Twenty in the first piece and 40 in the second."That tendency is well controlled by some kind of quantitative sense when the largest number is small, say less than thirty, but Art had trouble with it in other cases. For example, he was asked, if the number line from 0 - 100 were divided in half at fifty, which piece would be longer. "This one," he said, pointing to the 50 - 100 piece. Asked if it was a lot longer, he said, "No, they are the same size."

He and the interviewer worked on getting him to understand the questions correctly. After he did a lot of counting of individual blocks in the sticks, rearranging them into 10's, a twenty and then a thirty, he was asked:

Int. What did you find out?

Art. There's twenty here.

Int. Twenty what? (meaning unit cubes, ones, blocks, etc.)

Art. (No answer)

Inst. What if you put 20 and 20 together?

Art. It'd make 40.

Int. Forty what?

Art. Twenty. (Presumably, the forty is made up of twenties.)

Here we see a common occurrence, the child is still working out one communicational or conceptual difficulty while the interviewer is trying to get him to think of a new question.

The interview concluded with the game of "The Wizard and the Invisible Hand" in which Art was gaining competence. He also demonstrated that he could count backwards from ten to one.

In February, the fifth interview with Art again focused on counting. He counted two sets of 34 one-inch cubes very accurately, for example, managing to keep track accurately of which cubes he had

counted without touching them and which ones he hadn't, even when they were arranged at random. For example, when he was interrupted after counting 17 of the cubes, and asked to draw a line around those he had counted, he quickly drew a line which enclosed exactly 17 and then continued on counting to 34. At one point later he had made two rows of thirty-four cubes each and began to divide each row in two parts, counting one part of a row as thirty by mistake. (He had jumped from 19 to 30). When asked what had happened when he counted thirty, he replied, "Thirty, thirty, thirty, thirty." Evidently he was working on the problem of the previous interview, decomposing 40 into twenties, only he had mistakenly called 20, "thirty," and not noticed that there were only 14 in two of his four twenties. He tried to correct himself, counting 45 and 45 and later correctly counted 34 and 34.

Mathematics Pedagogical Dialogs and Instructional Policy

Ivan L. Ruiz

The pursuit of excellence in mathematics in this country is a concern that dates back to the Sputnik era when the Federal government, caught off-guard by the technological achievements of the Soviet Union, attempted to close the gap by creating programs to ameliorate the deficiencies in science, mathematics, and language arts programs in schools throughout the nation. More than twenty years have elapsed and this nation ranks below most industrialized nations in school math achievement. Language programs are in no better position partly due to the ethnocentric nature of American thinking concerning foreign language learning and as a consequence of the economic constraints being placed on bilingual education programs in the schools in light of an unstable national economy.

The thought of mathematics conjures in the minds of a vast portion of the American public negative images of instruction in the primary grades: instruction composed of rote memory and "religious" adherence to rules. These same images are also all-too-common in the minds of a great portion of school teachers. The circle is complete when we find large numbers of elementary and high school students lagging behind in mathematics achievement and possibly building a negative image of quantitative reasoning.

For ethnic minorities, the proportion of students floundering in math is higher than that of the Anglo population. The consequences of this fact are not to be taken lightly. In achievement-oriented societies such as this, school grade attainment is used as a mechanism of rewarding the successful with wealth and status while denying the same priviliges to those not as fortunate (Carnoy, 1974, Sennett, 1972, Banfield, 1970). As technological advances continue with

time, ability to communicate in mathematical terms becomes prime, and this is translated into mathematically-related skills required in a greater number of positions in the job market. Moreover (and this is not new) mathematics is an integral component a large portion of careers, from the hard sciences to the social sciences. Having less than a competent ability in mathematics precludes the possibility of participation by racial minorities (and women) in many lucrative and socially-rewarding careers.

In order to ameliorate the problem of math underachievement by racial minorities, several programs have been undertaken. Amongst these are Project SEED, Upward Bound, and tutoring and study services at universities. Some of these projects bring minorities engaged in math-related careers to the classroom in order to provide learners with role models and enhance personal self-concepts. In spite of these efforts however, only the tip of the iceberg appears to have been touched.

There have been calls for the aggressive federal funding once customary during the Johnson administration to alleviate similar problems, and charges that the federal government is retrenching when meeting the needs of racial minorities are proving very credible at the funding level. Private agencies and foundations are being sought more aggressively by those seeking funding for alternative educational solutions and programs. One claim is that the red-tape associated with government grants and programs is lessened with private funding sources. (It is customary for school districts to hire extra personnel to handle the paperwork required by the federal government in the funding of programs.)

Work at the University of Illinois

The work being undertaken by Dr. John (Jack) Easley and associates at the

University of Illinois is an attempt to delve into the problem of math education from "below". By this, I refer to the emphasis being placed on the classroom teacher as the principal actor in our endeavor to precipitate positive change in the traditional method of teaching primary school mathematics. The approach is novel in that policy is being affected from the "grass roots" level, more precisely, the classroom, rather than from the office of a superintendent who rarely has the occassion to witness the acts and actors in the classroom scenario. Moreover, research on educational innovation has pointed to the fact that when teachers have input in creating change, the innovation tends to have a higher survival rate than when change comes from "above" (Havelock, 1973).

In describing my observations of the innovation in in-service teacher education being introduced at the Steuben School in Kankakee, Illinois, I will draw from a paper by Michael Huberman (1979) which reviews the work being carried out in Geneva, Switzerland on "individual patterns of information search and use in implementing change at the school and classroom levels."

Huberman cites the Fullan and Pomfret (1977) review of the research on curriculum and instructional implementation where under the topic of "determinants of implementation", four categories are discussed: Characteristics of the Innovation, Strategies, Characteristics of the Adopting Unit and Characteristics of Macro-Socio-Political Units. I will describe the work at Steuben school along the lines of these categories.

1. Characteristics of the Innovation (explicitness, complexity)

A concern voiced by a majority of the teachers in the project was the matter of novel instructional approaches that work well at the in-service workshop, seminar or conference, but fail when put to the test in the classroom

setting. These same teachers also commented how the demonstrators of these techniques seem to have a degree of "magic" in their act, a kind of magic that is not easily transmitted to them.

Such was not the case in this innovation. Teachers seemed impressed by having someone in-situ who could help them solve teaching problems. The nature of the material (or techniques) presented them was not too far away from their realm of experiences. For some of the teachers it was simply a matter of "rearranging things" in order to simplify their teaching techniques. Prior experience seemed to help the innovation along. This is to say that the subject matter was not new -- they all had their share of trial and error in teaching mathematics.

As in all innovations, not all of the clients respond with the same degree of dedication and personal involvement. Two of the teachers who originally agreed to participate in the project were slow in becoming involved. The *RP's were as unobstrusive as possible in their approach. They would ask if the teacher wanted assistance and when the response was affirmative, the RP's would contribute at whatever point the teacher had stopped. The teacher was made a participant in the innovation process. She was encouraged to take notes and consult with the RP anywhere along the lesson. Remarkable gains appear to have been made if teachers' enthusiasm could be used as a measuring stick. The two teachers that could be described as laggards in the innovation process became more involved as they observed the happenings in the classroom of their peers. In one of the classrooms where the RP's felt somewhat uneasy and intrusive because of the extreme level of teacher control over the pupils, there came a surprise when the teacher remarked that she wanted a share of the kinds of things her colleague next door was getting in

*Resource Persons

her classroom. (She was under the impression that time constraints on that particular day were going to leave her without the RP's treatment).

The most prominent appeal that the RP's made to marshal support for their work was principally in the ideological domain, namely, that students of the kind being helped would respond well to the treatment. Of course, Hal Taylor's influence was of special significance. Here was another teacher with a genuine concern for students' welfare, demonstrating a method which had a prior track record of success! As one teacher remarked, "His is not a group out to test a technique fresh out of the university lab!"

Perhaps the most unique feature of the interaction observed between the RP's and the teachers was the concern the RP's showed for the teacher's opinion. It was pretty well demonstrated that the knowledge flow was a two way affair. The more the teacher told the RP's about the kinds of problems she encountered and the solutions she attempted, the more enlightenment both parties experienced in working out an alternative solution. The teacher was never made to feel that she was'doing it wrong.' It was quite apparent that a dialog, a pedagogical dialog was taking place (Freire, 1968,1973).

2. Strategies

An overriding concern of the RP team was to insure that whatever knowledge they had to impart must be workable in the classroom milieu. Teachers were not interested in time consuming methods that would set them back in their schedules and produce marginal results.

The RP's took every opportunity to elicit feedback from the participating teachers. The teacher's lounge was a place where teachers would respond to questions from the various resource team members and vice versa. As always, there is never enough time. For someone not in the school setting, the task of the teacher is a herculean one! The amount of time a teacher spends on

non-instructional duties is amazing!

Our task was not made any easier since the RP's visits to Steuben School were made on a Friday, a day when everyone is winding up for the weekend and wanting to get home as soon as possible. The more motivated teachers would stay after class to discuss the issues at hand. At least in one instance, the discussion was carried to a local "watering hole" where a majority of Steuben School teachers were present.

One significant goal of this project is to promote communication between teachers at the various sites. One of the ways this was accomplished was through the visit made to Hal Taylor's school in Michigan. It was felt that this kind of inter-communication would stimulate the participants to become change agents themselves within their school districts, giving additional credibility to the methods employed. In Huberman's terms, "Increased similarity of the model to the learner appears to decrease discrepency and thus facilitate implementation."

Perhaps the main incentive used to draw teachers to participate in the project was that RP's would be helpful in the classroom setting as opposed to offering help second hand via an in-service workshop. Other incentives which may have contributed to our high rate of participation was the matter of this being an innovation from below, rather than mandated from the superintendent's office, and that it was purely voluntary. There were few rewards here by way of released time or extra pay, and this was evidenced by the difficulties encountered in scheduling the trip to Michigan since the principal was not at first amenable to the idea of letting more than two or three teachers go at one time. If there were any rewards, they were intrinsic in nature, e.g., improvements within the classroom. However, it must be mentioned that mere participation in the project may have ascribed to the participants a "special" kind of status among the teachers at Steuben (Easley, comment).

3. Characteristics of the Adopting Unit and of the Macro-Sociopolitical Units

The Steuben School is located in central Kankakee, Illinois, a few blocks from the downtown business district. The facility has been transformed into a "magnet school" with special programs such as two Montessori classes used to retain the Anglo sector of the population who in the past had been migrating to Bourbonnais, a suburb bordering Kankakee to the northwest. While the area surrounding the central business district of Kankakee is becoming increasingly Black, the neighborhood around Steuben school is mixed. Casual discussions with both Anglo and Black residents of the area revealed that both groups are quite content to live side by side. Some parties responded that this was the way it was for as long as they remember. Teachers at Steuben School reported that the community is supportive of it's programs.

The social climate within Steuben School is both stable and warm. Principal Roger Rainbolt appears to be quite concerned about the happenings within his building (several teachers reported that he makes classroom visits on a regular basis), and is receptive to outsiders introducing changes that may effect positive results in the student body.

The teachers give the impression of being a fairly well-knit group of hard workers who chat frequently in the teacher's lounge. All showed a concern for the welfare of the students and expressed their commitment to do the best under the circumstances. This concern was seen when two of the teachers contacted Dr. Willie Davis from the district superintendent's office so that he could come and visit us. Their interest was in seeing what the Kankakee School District could do to fund extra meetings of our project during the summer and coming academic year. This kind of enthusiasm on the part of the teachers showed for one, that the innovation we were attempting seemed worthwhile enough

for the clients, to merit funding, and two, that innovations can be initiated from the school room to the district level.

Further contact by the writer produced a meeting with Dr. Keesee and Dr. Davis at the district office on May 1, 1980 for the purpose of discussing funding by the Kankakee School System. Once again, the project met a warm reception. The matter of federal funding for our project was brought up and this elicited a pessimistic response from Dr. Keesee since he has been told to go to the local level for funding of this kind when he approached funding agencies in Washington. Additionally, Dr. Keesee mentioned that for the majority of federal grants, the funding cycle deadlines had passed in December or early January. Dr. Keesee also made mention of Title IV monies that the State of Illinois had, but in his words, "I wouldn't even try for those," implying that most of the monies had been earmarked at this late juncture. He did bring up one possibility, NIE's "Knowledge Use and School Improvement Grants Program" whose filing deadline was less than a week and a half away. Dr. Easley tackled the job of putting together a small-grant proposal and we expect to hear from the National Institute of Education in late September. Meanwhile, Dr. Keesee assured us that they would make an effort to partially fund our work if additional funds came to the district from grants or other sources. This, of course, is very tentative.

In summary, the sociopolitical ambiance within the school and in the community appears rather conducive to the kind of change being attempted. Although the writer was made aware of racially-motivated strains that have been developing within the respective communities in recent years, both sectors have an interest in common, namely, to see their children achieve a good education in the schools. Strong administrative support is evident at the school and district levels. Dr. Keesee and Dr. Davis offered their good offices

in helping us get school board approval should additional funding be forthcoming to allow expansion of our services to other schools.

On Policy, Implementation, and the Classroom Environment

The terms policy and implementation have acquired a special meaning in American society during the past two decades. The street wars that marked the sixties and early seventies (and for similar reasons in the eighties when the Miami Riot is taken into account) were a result of the disenchantment of the nation's youth towards U. S. war policy and of the racial minorities awakening to the realization that American institutions were following and implementing policies that fostered inequality. Thousands suffered maiming and death at the hands of our national guard and local police who in turn responded to other policies resulting in orders to "shoot to kill or maim." "Kill" or "Change the System" were the battle cries of the day. Again, killing or changing the system mean the altering of policies by which the system functioned.

The plethora of civil rights legislation that followed the turmoil, put into effect policies and implementing regulations that attempted to ameliorate the excesses that kept all Americans from ejoying the same rights or priviliges regardless of color, race or national origin. Racial and national minorities looked upon Capitol Hill as the champion of civil liberties and equal opportunity for all.

Interestingly enough, policies that emerged as a result of the civil rights movement of a few years back offer the student of government voluminous material from which to research the intents and outcomes of government mandates. Going back nearly three decades is the landmark Brown decision which had the effect of "terminating" school segregation in most districts across the land. Amazing as it may seem, the Brown case has been revived this very year in Topeka,

Kansas, because the remedies the court mandated in 1954 have not as yet been carried to completion there! In Chicago, an industrialized northern city, the Board of Education is being sued by the federal government for its refusal to integrate the public schools.

As anyone filing a complaint with the Office of Equal Employment Opportunity at the state or federal level soon discovers, the backlog of unresolved cases is monumental and the waiting period for a resolution usually extends to years. Giving Congress the benefit of the doubt, it is unlikely that the outcomes cited were intended at the outset when the legislation formalizing established policies was made.

The discrepency between the intended policies and actual implementation of those policies is the subject of much research today. Richard Elmore is among the most recognized scholars in this area. In his paper, "Complexity and Control: What Legislators and Administrators Can Do About Implementation" (1979), Elmore points to the dichotomy between control and complexity:

> "The traditional devices that legislators have relied upon to control policy implementation -- more specific, tighter regulations and procedures, centralized authority, and closer monitoring of compliance -- probably have the opposite effect that they are intended to have. Rather than increasing control, they increase complexity. And as complexity increases, control itself is threatened." (p.2)

Elmore postulates that programs may be operating as intended in the "regulatory" sense, e.g., adhering to rules of financial accountability, but failing in the "programmatic" sense, that is, in the delivery of the intended service. Elmore further argues that legislators and higher echelon administrators may have the power of rule-making but rarely can they effect changes in the quality or capacity of the services. He adds, "Quality depends mainly on the skills and competence of the people who actually deliver the service, and only secondarily on their compliance with rules and regulations." (p.10)

Giandomenico and Wildavsky (1977?) posit that there is an symbiotic relationship between implemention and policy, essentially saying that a change in one affects the other. They term this "evolutionary implementation" where behavior is not prescribed but given "ranges of tolerance for policy changes and adaptations." "Faithful implementation," they add, "no longer means a one-to-one correspondence between policy idea and policy result, but only that certain boudaries drawn around the central policy core have not been overstepped in the process of development." Giadomenico and Wildavsky conclude:

> "There is no need to feel guilty about failing literally to carry out a mandate inherent in a policy because there is no such thing. Unless the matter is uninteresting, i.e., pre-programmed, untouched, as it were, by human hands or minds, a policy cannot contain its own consequences. Implementation must be evolutionary; it must reformulate as well as carry out policy."

The writer has few qualms with these formulations, however, Giandomenico and Wildavsky fail to address the matter of the value orientations on the part of the implementors. And since that is a factor that cannot be practically controlled, care must be taken when drawing up the parameters by which policy implementation will be guided. One need only be reminded of the kind of implementation of civil rights policy that would have ensued in the South, had it not been for the "militaristic" kinds of mandates that the federal government had to hand down to the states in order to insure that the spirit of the law was being carried out.

It is clear that the field of policy and implementation is yet to come of age. Meanwhile, regardless of the philosophical or theoretical points of departure that social policy and consequent implementation may take, one thing is certain, <u>people are being affected</u>. In our case, these are the children in the classroom who are not fairing very well in mathematics. It is my contention as well as Easley's (1980) that the classroom environment

is the final stop, and perhaps the most important one in the long journey of policy implementation in education.

The Classroom Environment

It is intruiguing how educational policy makers frequently edify complex "master" plans, sometimes down to the last detail, but fail in one crucial step, namely, the taking into consideration of the impact a given policy may have on the classroom environment and more precisely, on the teacher.

We all carry images of our childhood days in elementary and secondary schools. And invariably, we all remember with pleasure a special teacher we may have had in one of those grades. Likewise, negative feelings and images may be present when we reminisce on the experiences we had in a classroom with a teacher we disliked. The classroom environment is perhaps one of the most complex phenomena in the educational enterprise. Surprisingly enough, it is one of the least studied.

Within the confines of a few yards of space, the classroom is a microsociety within the larger context of the school and surrounding community. The teacher, regardless of his or her desires, ends up being the principal social control agent in the classroom. This implies the parcelling out of rewards and punishments, grades, and increasingly, the delineation of the kinds of physical behavior that will be allowed in the classroom setting. The teacher is "big medicine"... powerful and indispensable!!

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Interview Report of
In-Service Teacher Training Project
in Math Education
at Steuben Primary Center
Kankakee, Illinois

Carol Chapelle
University of Illinois
June 12, 1980

The Purpose of the Project

The project described was born out of the general concern for the lack of communication that exists between educational researchers and classroom teachers as well as the specific problems that this deficiency causes in the area of elementary math teaching. In order to address this problem, the project's purpose in Steuben Primary Center in Kankakee, Illinois was an interchange of information and experience for the benefit of both parties: the math education researchers from the University of Illinois and the classroom teachers at Steuben. This report includes the dates and nature of interaction between educational research people (RP's) and teachers, comments from both groups as well as from those teachers at Steuben who choose not to participate in the project, and finally, some concluding points. Examination of these aspects of the program makes it clear that to a great extent information and experience were exchanged to the benefit of both the RP's and the teachers.

The RP's

The RP's consisted of a group from the University of Illinois: Dr. Jack Easley, Dr. Bernadine Stake, Ms. Aimee Grieb and math educator, Mr. Hal Taylor. Jack is a professor in the department of Elementary and Early Childhood Education and has long been associated with Bernadine, who received her Ed.D. from that department. Aimee is a new graduate student in education and has also worked with Jack in the Sciences Tutoring Laboratory. Hal, a former student of Jack's, is currently the director of the math lab for Lincoln Consolidated Schools, Ypsilanti, Michigan. The RP's share a common philosophy in their work: teachers must trust and feel comfortable with RP's before any math work will be accomplished and establishing a rapport takes time. Throughout the semester, the spiral of trust in the RP's and accomplishment in math teaching seemed to be growing.

The Calendar

1979 December	Two papers, described below, were distributed to the staff. They were asked to look them over before the January 9 staff meeting.
1980 January 9	Jack spoke at the staff meeting to invite questions about and participation in the forthcoming program. By the end of a week, six teachers had volunteered.
January 20	Jack and Hal met with teachers, Betty McMenamin and Susan Shadid for dinner. All of the teachers volunteering had been invited.
January 21	Jack and Aimee worked with individual students because teachers had identified slow children as their most immediate concern. Hal taught in Betty's and Susan's classes.
January 25	Aimee worked with teachers and students.
February 1	Jack and Aimee worked with teachers and students.
Febraury 8	Jack and Aimee worked with teachers and students.
February 15	Jack and Aimee worked with teachers and students.
February 22	Jack and Aimee worked with teachers and students. Hal taught in Betty's, Bea Kennedy's and Mary Vaughn's classes at their request.
February 29	Aimee worked with teachers and students.
March 7	Aimee worked with teachers and students.
March 14	Jack and Aimee worked with teachers and students. Hal taught in Polly's Susan's Alice's and Beth Atwood's classes at their request.
March 21	Aimee worked with teachers and students.
March 27-29	Conference in Peoria.
March 28	Aimee and Bernadine worked with teachers and students.
April 11	For reasons of equity and continuity, the teachers were informally divided among the three U. of I. RP's: for the rest of the semester Jack worked primarily with Alice and Susan; Aimee with Kim and Polly; and Bernadine with Beth and Martha Judson.

April 18	Jack, Aimee and Bernadine worked with teachers and students. Hal taught in Polly's, Susan's, Alice's and Martha's classes at their request.
April 25	Jack, Aimee and Bernadine worked with teachers and students.
April 27-28	Visit to Ypsilanti
May 2	Jack, Aimee and Bernadine worked with teachers and students.
May 9	Jack, Aimee and Bernadine worked with teachers and students.
May 16	Jack, Aimee and Bernadine worked with teachers and students. Hal taught in Polly's, Susan's, Alice's and Martha's classes.

Description of Interactions

Presentation of the Project to the Staff

In December, Jack met with Roger Rainbolt, the principal at Steuben, in order to discuss the project and to give him two documents to be distributed to the teachers and read before the staff meeting on January 9. One of the articles included an enumeration of ways in which RP's might be helpful to teachers (Appendix A). The other, a substantive math article, discussed problem solving in elementary math. When Jack appeared at the staff meeting, he spoke for fifteen minutes about the proposed project then invited questions from the teachers. There were none. At that point there were six volunteers who said they would be interested in working with the RP's: they were the same six who had had previous encounters with the U. of I. team. Beth, one of the six, indicated that the teachers already knew that the RP's were very helpful to have in the classroom so there was no question that they wanted to continue.

Conferences with Teachers

Conferences with teachers typically serve two purposes: pre-conferences are used to identify areas of concentration for the forthcoming lesson; post-conferences are to highlight important aspects of the same lesson and to identify possible directions to follow in subsequent lessons. Both types of conferences are as unique as the people participating in them. For example, Hal often has only a few minutes before the lesson to learn from the teacher what he's going to do in a particular lesson. Aimee, on the other hand, calls Betty each week several days before her visit because they have found that to be the most comfortable arrangement. Jack and Bernadine and their teachers prefer to meet ten or fifteen minutes before school to go over their ideas. All of the pre-conferences have one thing in common: the RP's ask the teachers what they want them to do that day. "And teachers know what they want us to do," Hal pointed out.

The post-conference generally takes place during the teacher's break, lunch or after school. This conference cannot be characterized by one formula any more than the pre-conference can: the RP and teacher discuss what they believe was important about the preceding lesson. Hal emphasized that he tries to discuss possible follow-up lessons to the one that he taught and to look in the teachers' text books for supporting materials.

Jack indicated that the quality of interaction in these conferences was at varying levels of sophistication. Some teachers seemed to feel comfortable about disucssing their math teaching problems with the RP's while others did not yet want to be as open with them. "Their trust grows, but it's slow," Jack explained, "We have to give them time."

RP's Helping in Class

After the teacher has identified what the RP's are to do in class in the pre-conference, they do it. The assistance that teachers ask for generally takes two forms: working with individuals or small groups of children outside of class, or teaching the class as a whole.

At the onset of the program, most of the teachers identified their areas of concern to be individual children who were unable to keep up with the rest of the class in math. In these cases, the teachers indicated the kinds of problems the children were having and the RP's addressed them in their time together. As the semester progressed, in the individualized classrooms (Kim's and Beth's), the RP's continued to be asked to work with individual children or small groups while the other teachers began to request that the RP's work with their whole classes.

Similarly, when RP's are asked to work with whole groups, the teachers decide which topic should be pursued and how much they want to be involved in the lesson. For example, some of the teachers like to team teach with the RP's; sometimes they simply aid in class; other times they sit and watch, perhaps taking notes. It can't be siad that there is one best way for RP's to interact with the teacher in the classroom; the key lies in the teacher's feeling comfortable in choosing the position that is most beneficial to her in a given situation and being able to change that position whenever she wants to.

When Hal teaches, there is a kind of energy in the classroom that is not common in elementary classrooms. The children are excited; they're thrilled; they can hardly wait to speak - about math! "I try to keep the kids excited so that the teachers can see that it can be done," Hal explained. "I

keep math suspenseful and exciting by not telling the kids the answers. I make them figure them out." It must have been exciting; Hal began the semester by visiting two of the teachers' classes. By the end of the semester, his talents had been requested by all of the participating teachers.

Hal was able to enumerate several factors that contribute to his success as a math educator: he practices a lot; he knows math; he knows children; he doesn't have a rigid pattern in mind when he begins a lesson. While it is not likely that a combination of these factors in any teacher would result in another Hal Taylor, it is reasonable to suggest that a person who attained them might be a good math teacher.

Out-of-Town Conferences

There were two out-of-town gatherings available to the RP's and participating teachers. Jack, Susan and Polly attended the three day conference in Peoria, Illinois on March 27-29 sponsored by the National Council of Teachers of Mathematics. There they found interesting talks, exhibits promoting materials and demonstration tables. The three of them also had the opportunity to meet informally during the weekend to discuss relevant math issues as well as to get better acquainted with each other.

Casual discussions were also an important aspect of the two-day sojourn to Ypsilanti on April 27 and 28 to visit Hal's math lab and classrooms. The trip was made by four of the teachers: Kim, Alice, Beth and Martha. After seven hours of informal car talk, they arrived in Ypsilanti, where they visited the math lab to inspect the materials available and to watch as Hal demonstrated some of them. That evening they took a break for more socializing at Hal's house, then dinner. Monday morning they visited various math classes and a teachers' store. In the car on the way home that night, they had another

seven hours to reflect on and share what they had experienced and how it might affect them. Several of the teachers cited this as their most valuable experience of the semester.

Establishing Rapport Between the RP's and Teachers

Because of the trust that is prerequisite to the accomplishment of more substantive objectives, establishing rapport between RP's and teachers was considered of utmost importance. To this end, numerous informal gatherings were attended outside of school other than those mentioned in conjunction with the two trips. For example, the semester began with a dinner invitation to the teachers. Also, it was not unusual for the teachers and RP's to go out for a drink after a day of work together. These gatherings were of great value in establishing a successful working relationship between the two groups.

The RP's Reactions to the Activities

Even though the activities were teacher initiated, at the onset of the program it was proposed that both teachers and RP's might benefit from such a program. The RP's were able to give only a little evidence that they had seen teachers using what they had learned. For example, Aimee said that one of her teachers had made a number of realizations as a result of working with her; one was that a perfectly ordered (disciplined) class did not mean that learning was taking place. Also, Jack indicated that one of the teachers was using some of the material that she had learned from Hal and him. However, the overall attitude of the RP's was "these things take time."

On the other hand, the RP's were enthusiastic about what they had learned from the teachers. Although Jack has been working with children's thinking

and math for years, he said that he was "pleased and surprised about what he had learned" by working with these teachers. For example, he explained that he had learned from one of the teachers that there is a conflict in children's minds between operations using counting and those using place value. Also, he encountered some children in one of the classes who didn't understand that large numbers cannot be subtracted from smaller numbers. He has worked on solutions to these real problems and will be able to share what he has discovered with other educators.

While Jack's realizations will benefit math educators particularly, Bernadine pointed out some insights she had attained that would be relevant to teacher training in general. She elaborated on the importance of understanding what teachers are thinking, their expectations of children and their perspectives on teaching. "You have to know what they are thinking to work with them effectively," she insisted, "just like we believe that teachers have to know what children are thinking to teach them math." Bernadine believes that she has come to understand teachers better as a result of working closely with them at Steuben. Thus, she has become a more effective teacher trainer.

Comments from Individual Teachers

Participating Teachers

Susan Flanders. Susan, a first grade teacher, was familiar with the U. of I. team before the introduction of this program because they had been interviewing two children in her class the year before. She decided to continue working with them because she knew them.

This year she had Hal teach in her class, taught with Jack, and attended the conference in Peoria. Classroom teaching experiences were preceded and

generally followed by brief conferences. She also engaged in several informal discussions with RP's outside of school. Susan had difficulty isolating one aspect of the program as most helpful. "It has all been very helpful," she commented, "because you are exposed to new ways of doing things that you've been doing the same way for years." The RP's always asked her what she wanted them to work on and then proceeded to make noticeable improvements with the children. For example, her children saw addition and subtraction as totally unrelated processes until this subject was addressed by the RP's at Susan's request.

Susan had two regrets about the semester's work: she would liked to have had more time at the Peoria conference; she wished the cuiseinaire rods had been introduced to her class earlier in the semester. With these her only disappointments, Susan indicated that other teachers would probably be interested in a similar program. "I think any teacher, if she's honest, will say that she can use all the help she can get." She also noted, however, that there will always be those teachers who don't want to be bothered, or who feel that they have their own way of teaching and don't want to see anything else. Nevertheless, "Any teacher should realize that having two adult bodies in the room makes teaching easier."

Martha Judson. Martha is a second grade teacher who had also worked with the RP's on other occasions. She saw this project as a continuation of the earlier one and therefore remained a participant.

Bernadine and Hal worked with her whole class and Bernadine and Jack worked with individual children that Martha designated as needing extra help. Classroom visits were typically preceded and followed by brief conferences. Martha also went on the trip to Ypsilanti, an excursion that she is very grateful to have been a part of. In fact, she cited the Ypsilanti

trip as the most useful thing that happened all semester. "It's easier to think about school when you're not under the pressure of the clock," she explained. She felt that the school day was too short and busy to really reflect on specific issues of math teaching.

Generally, Martha enjoyed working with the RP's. She perceived them to be a cooperative group that always asked what she wanted them to do and when it would be convenient to have them come. She said she had gained a lot from the program in the form of ideas, methods for use of manipulatives, and materials she would like to use. The new approaches have made it easier for her to teach.

Martha indicated that a similar program might be well received in another school if it's introduction consisted of a clear proposal of what kinds of activities were to take place and to what end. She pointed out that it was probably not a coincidence that the teachers who were working with the RP's this semester were the same ones that had worked with them previously.

Beth Atwood. Beth teaches second and third graders in an individualized classroom. She had also worked with the U. of I. team before this semester and felt that this was simply a continuation of earlier work.

Because Beth's is an individualized class, the RP's generally worked with a few children that Beth specified as needing extra attention, either remedial or advanced. On one occasion, Hal caught the class as a whole because Beth had requested a demonstration of the dienes blocks. She also went on the visit to Ypsilanti.

The assistance given to individual children was isolated as the most helpful aspect of the program. "I've seen tremendous progress," Beth

reported on two of her slow children that Aimee had been working with. In Beth's class each child is busy working on something so she is free to circulate and observe what the RP's are doing. Her observation of the RP's has resulted in her learning several things that she can incorporate next year. For example, she would like to use the methods she has learned with the dienes blocks and the cuisenaire rods. In fact, in order to pursue these methods, Beth pointed out enthusiastically, "Hal is going to be available to work with us for two weeks this summer.

Kim Tenney. Kim's class is similar to Beth's; it is an individualized class of second and third graders. Like the others, Kim had worked with the RP's before, however, she saw her involvement this semester not simply as a continuation of previous colaboration but as a desire to continue learning from the RP's as she had in the past. She felt strongly that the work she had done with the RP's in the past few years had really helped her to understand how children reason; this has helped her teach math.

This semester, the RP's generally worked with the individual children that Kim specified on the topic she indicated they needed help with. Hal taught her whole class once and talked to her briefly. She went on the trip to Ypsilanti.

Although Kim was very happy for the assistance she was given with the children, she felt that she wasn't really growing professionally as much as she might if the RP's took a more definite position about what they were doing. For her own development, she would prefer to see more direction on the part of the RP's. For example, she thought it would be more helpful if they (1) asked for a child with a particular problem, (2) tried a method believed to address that problem, (3) shared successes and failures with the teachers.

In order for a similar program to be successful in another school, Kim believed it would be necessary for the team to communicate their purpose to the teachers who would potentially be involved. "I really didn't know what they wanted to accomplish this semester," Kim admitted.

Alice Wiess. Alice teaches third graders in a traditional classroom. She had previously worked with the U. of I. team but she joined this program specifically to get help with some of the slow children in her class.

The RP's began by working with the slow individuals that Alice pointed out. When these children had been brought up to the level of the class, Alice asked the RP's to work with the whole class. Her eyes lit up and she smiled as she described how the RP's handled her class: "First one starts out and he'll do something while another sits back and then says, 'I'll use another approach,' so the next trip around he'll say, 'OK, I'll take it,' and then the next one does the same thing but they're all using different approaches - and they were pretty good." She particularly liked to see them work on the same things that she was working on in class so she could see them done different ways. She also commented that she has been able to use some of these approaches herself. For example, at the end of the day when there are about five minutes left, she sometimes gives them the same kinds of oral problems that she has seen Hal use.

Alice felt that teachers in other schools should be interested in a similar program, if for no other reason than simply to help them with their slower children.

Polly Langlois. "There must be an easier way of doing this," Polly summarized her feelings on math teaching at the time she was approached with

the possibility of participating in this math project. She highlighted two things in particular that she was having trouble teaching: story problems and operations using several digit numbers. Her situation had already forced her to seek some alternatives: she had changed math from afternoon to morning so she would have time to follow up with remedial work in the afternoon if necessary.

The RP's began to work with Polly's third grade class on the areas she specified as problems. She team taught with RP's and observed RP's teaching her class. Each visit by an RP was preceeded by a phone conversation early in the week and followed by a half-hour conference. (Polly's break was the half-hour right after math.) Polly attended the conference in Peoria, where she found a session on the very problem she was having with subtraction of multiple digit numbers.

A few things were noted as especially helpful to Polly: use of squares for teaching multiplication and division, introduction of "can't do" problems to emphasize the fact that large numbers cannot be subtracted from smaller ones, use of unifix cubes and dice with no numbers. "They just love those games!" Polly explained in a tone that indicated that the children were not the only ones who loved the games. She commented that the children had improved a great deal, and even though conferences with RP's often took her break time, she felt that it was well worth the time spent in the long run for both her and her children. In fact, she said that she wished that the RP's could have come more often.

Polly said she hoped that teachers in other situations would be interested in a similar program because even though teachers could purchase new materials, they wouldn't be useful without knowledgeable people to teach them how to use them.

Non-Participating Teachers

Karen Johnson. Karen, who is working with kindergarten through second grade in a Montessori program, had had some positive experiences with the U. of I. team prior to this program when they worked with some of her students individually. However, this is the first time she has been involved in the Montessori program and she not only had her hands full with it but also was very satisfied with the way the children were learning math with the methods provided. In short, she didn't see the need for help with math teaching in her classroom but recognized that in other classroom situations it would be desirable to have such outside help.

Janet Billings. Janet's situation is quite similar to Karen's; she is working in a Montessori classroom for the first time this year with kindergarten and first grade and had previously had some experience with the U. of I. researchers. The people who were advising her in Montessori methodology suggested that she concentrate on it exclusively for the first year. Nevertheless, "I regret not getting involved with the U. of I. team this year," Janet admitted as she reflected on the semester. In fact, Janet was able to take advantage of the researchers working at Steuben by asking them to work with a couple of kids who were having trouble in math. Although, like Karen, Janet was happy with the Montessori approach and didn't see any particular problems with it, she pointed out, "A lot of times you don't feel that you have a problem until you are shown an easier way." It was the belief plus the fact that the RP's offered practical help that caused Janet to feel that teachers in other schools would respond favorably to an invitation to participate in this kind of program.

Patty Williams. Patty, a first grade teacher, was another who was pre-occupied with a project that kept her from working with the RP's in math this semester. She is a reading specialist, formerly with the Title I program, and was continuing her reading research with her class. Because of that work and the fact that she was new at Steuben this year, she felt that her schedule would not comfortably allow her to take on any more. However, because of some of the things Patty heard from other teachers about the RP's (e.g., they helped in Polly's class; they're not eccentrics), she had second thoughts about the benefits her class might have received from them. Interestingly, when Patty's attention turned from the time the RP's would take from her toward the help they could bring to her class, she was able to enumerate several areas in which she could use assistance with math: clarification of the processes of addition as compared to multiplication; remedial work with slower individuals because there are so many of them that they have a severe impact on the group as a whole; use of expertise in setting up more "math labs" in her room, some of which she has already established; more individual attention to each student. Patty plans to work with the RP's next year if the opportunity is still present.

Sally Anderson. Sally, like Patty, is a new first grade teacher at Steuben this year. "I really didn't understand what it was all about," was her response to the research project. Sally said she didn't feel comfortable enough with the practical aspects of the program to get involved; she didn't know what the RP's would do, when they would come and how much of her time they would take. Even though she believed she could use some help with some of her children who are not being reaching in math and she had heard from teachers who were working with the RP's, Sally would want to

know more about the details of the project before she would seriously consider spending her time on it in the future.

Sandra Selinker. "Who is that masked man?" Sandy was referring to Jack, who had been appearing in the halls at Steuben "for what seems like five years." She never learned anything about the math project because she was under the impression that the teachers who were working with the RP's had been personally invited to do so. Since she hadn't been invited, she didn't pay any attention to them. Sandy said that she didn't have any trouble teaching math but that she had some students who had a lot of trouble learning it. She didn't seem to feel that any outside help would have been beneficial to her or her students.

Summary of Salient Teacher Comments

Why Did (Didn't) Teachers Get Involved in This Project?

Almost all of the participating teachers cited one major reason for getting involved: they knew the U. of I. team to be personally pleasant to work with and helpful to have in their classrooms. Initially, several teachers intended to get assistance with individual problem children and only one cited problems she was having teaching math. Three of the teachers who chose not to paritcipate had other professional commitments that would not allow them to spend time and energy in this direction: two were initiating a Montessori program; the other was pursuing her own research in reading. The other two teachers who didn't join didn't fully understand the program: one felt the options available to the teachers had not been articulated clearly enough; the other felt that she hadn't been invited to join.

What Did the Teachers Like Best and Least About the Program?

A synthesis of all the teachers' comments indicates that they like to learn new ways to teach old concepts. As elaborated above, this learning can be accomplished in several different ways depending on the teacher's situation: RP's working with individual students or whole groups, brief conferences with teachers, informal gatherings and educational trips. One of the teachers pointed out long weekend meetings as her favorite because she felt that only during longer sessions, not during the school day, did the teachers have the opportunity to reflect on math teaching.

Teachers are less fond of activities in which they learn nothing. For example, one teacher said that she would be happier if she could have learned more about what the RP's were doing when they worked with her children individually. Overall the teachers felt they had learned a great deal by working with the RP's and were therefore quite satisfied with their experience.

How Did the Teachers Feel About the RP's?

"They always came in and asked me what I wanted them to do." This quote could have been taken from any one of the participating teachers. Thus, it can be said that the participating teachers found the RP's to be accommodating and flexible. It should also be mentioned that Steuben is a school with a history of participation in experimental programs. Consequently, the presence of extra adults in the classrooms is quite normal; it doesn't bother anyone. The non-participating teachers had a wide range of feelings about the RP's; one didn't know who they were or what they were doing there while, on the other end of the spectrum, another knew the RP's well enough that she was able to use their services with her slow children.

Would a Similar Program be Successful at Another School?

There was one salient response to this question: the initial presentation of the program must give the teachers a concrete picture of what they will do if they decide to participate, when they will do it, how much time it will take them and what they will get out of it. It was probably not a coincidence that the same people who already knew the U. of I. team were the ones who volunteered for this project. This might indicated that the rapport that had been established between teachers and RP's was a more important factor relating to involvement in the program than the invitation given at the staff meeting. The teachers felt that in a new school, a clearly defined program would be essential in the absence of familiarity with the staff. After the initial step, the Steuben teachers thought that other teachers would leap at the opportunity to get math help in their classrooms.

The Accomplishments

Even though the RP's had some trouble identifying evidence of great success with the teachers and the teachers, in many cases, couldn't say exactly what the RP's were doing there, comments made by each group about itself indicated that both parties had benefitted from their work together. The specific work done this semester was very important to the teachers and children at Steuben and the RP's from the U. of I. However, the model of this in-service teacher training program is of greater significance to education in general because it created a forum for the enthusiastic exchange of information and experience, which resulted in professional growth for both the RP's and teachers at Steuben and could do the same at other schools throughout the nation.

School Options for the Use of a Team of Resource Persons in Primary Grade Mathematics Teaching

Prepared by Jack Easley
Bureau of Educational Research
and
The Committee on Culture and Cognition

This document describes the situation at present in an emerging but informal project (or coalition) involving a variety of persons who in different ways seek to improve their opportunities for helping, and understandings of how to help, teachers in primary schools with the teaching of mathematics. The project will involve mathematics educators and mathematicians, who are permanently located in central Illinois and Southeastern Michigan, and a number of other well-qualified persons who are temporarily located in these areas, e.g., graduate students in mathematics education and visiting University faculty. All of us are impressed by the generally ineffective state of communication between mathematics educators and teachers. There is reason to believe that this state is partly due to the tendency for communication efforts to be seen as one-way, with the teachers only on the receiving end. We have thought of calling ourselves "resource persons" (RP for short), in spite of the fact that a one-way communication flow is still implied, simply because we haven't found a better term. "Collaborators" doesn't adequately describe the responsibility of the group to be helpful to those who have the responsibility to teach children. Other common terms, like "advisors," "specialists," "in-service educators," and "university teachers," suggest too much the expectation that we know what it is that teachers should be doing. Most of us only know what we would do if we were the teacher, which we aren't. Some of us don't even know that. We would, however, like to help teachers solve difficult and very significant problems which they identify or at least genuinely recognize as theirs. To maintain the one-way stance is to

be not taken seriously by the majority of teachers and to be ineffective in improving educational opportunity in the majority of schools.

We may state our goals as follows:

Goals: (1) RP's need to learn to avoid redefining teachers' problems in terms of their own special interests and theories so that they lose teacher interest. Positively speaking, we need to become much more sympathetic and sensitive to the difficult position in which most teachers find themselves as a consequence of the expectations society has regarding conditions and achievements.

(2) Teachers need to learn to avoid becoming overly committed to details of textbook or classroom procedures which don't make sense to children.

We adopt the following premises:

(1) Primary school mathematics can be made perfectly intelligible to children.

(2) Arbitrary rules and procedures are difficult to learn and destroy teacher and pupil interest.

(3) Resource persons need to learn how to listen to teachers between the lines and sense their needs and how to respond to them.

Jack Easley (University of Illinois, Urbana/Champaign) and Harold Taylor Remedial Math Teacher, and Math Lab Director, (Lincoln Consolidated Schools, Ypsilanti, MI) are interested in forming a long-range resource-person training team. One purpose is to discover the most effective ways of helping teachers. Another is to provide a variety of settings in which graduate students, and post doctoral fellows can learn new, more effective approaches to in-service work by being part of a research and development team.

Others who have expressed an interest in being involved include mathematics professors, Peter Braunfeld (University of Illinois) and John Ginther (Eastern Michigan University).

Graduate students and post-doctoral participants at the University of Illinois who have been involved include: James Kau (former elementary school teacher and principal from Taiwan; Bobbie Flexer, Math Education, University of Colorado, Boulder; Aimee Grieb, Math and Dance major; Ted Nicholson, Mathematics Instructor from Bethuen-Cookman College; Linda Brandau, former teacher from Bloomington; Bernadine Stake, former elementary teacher.

Schools in which significant preliminary work in a variety of approaches has already taken place include Kankakee's Steuben School (Primary Grade Center), Urbana's Thomas Payne School (Elementary), Bloomington's The New School (Elementary, private), and Ypsilanti's Lincoln Consolidated Schools (Elementary).

The research is currently supported by the Committee on Culture and Cognition, The Bureau of Educational Research, and the Sciences Tutoring Laboratory of the College of Education of the University of Illinois at Urbana/Champaign. The schools and their faculties and children have also contributed a great deal of time and, in some cases, a little money.

The following options for the use of RP's in primary school mathematics classes have already been tried and are working in at least one classroom:

1. RP works with a teacher on a problem identified (initiated) by teacher (e.g., borrowing and carrying, mental arithmetic, story problems, using math to teach discipline and discipline to teach math, etc., etc., etc.) RP may work with teacher in conference, may prepare new ideas in the form of worksheets, or other materials, or lesson plans, or otherwise, may work in the

classroom with the whole class (teaming with the teacher). One difficulty in describing the option is that teachers keep thinking of new problems while still working on old ones. This has to be OK, except the RP's may need to learn how to work on more than one problem at once. After all, that's what teachers have to do.

For example, a teacher brought up the problem of teaching problem-solving rules: What rules to teach that would help children faced with story problems to decide what operation(s) to perform on the numbers given. Working together four or five times over a period of nine months or so, the teacher and the RP have identified several steps to take: some additional work in computation skills (adding tens) and a new technique, making two- or three-column tables from story problems. The adding tens relates to regrouping. so they have started talking about that problem now, too.

2. RP works with an individual child or group of children whom the teacher feels need some new approach. Tape recordings of the sessions permit the teacher to work with the rest of the class at the same time. Teacher listens to tape recordings, selects what things to try, reports back to RP at next meeting (a week or two later) on what worked, what didn't work, perhaps making a tape to share with RP and posing new challenges for RP's next visit. Frequent, short visits between teacher and RP are required, although some of them might be accomplished over the phone.

For example, a teacher who is being observed twice a week in her two math classes by the same resource person and one RP who comes in about once a month, singles out kids for help. The RP's sometimes help children and sometimes just observe. The teacher talks with the RP(s) after school for an hour or so about the classes, what she is trying to do with particular

children, why she can't teach the way she would like to, etc., etc. She has a variety of materials but still the children are very dependent on her to tell them when they are right. She is trying some ways to get them to check their own work but the children don't take to it well.

3. RP works with entire grade level to build needed skills, (e.g., computation, problem solving, concepts) actually teaching all classes once a week with teacher in attendance and assisting individuals who need help. This is carried on over a period of 3 - 9 months in order to achieve noticeable progress in skill on tests or preparation for the next grade. Teachers adopt or adapt what they like of the RP's lessons to their own teaching.

For example, an RP is building children's familiarity with the doubles of the first twenty counting numbers, then getting them to use that knowledge to figure out quickly examples like 3 + 4, 7 + 8, 6 + 8, 8 + 7, e.g., doubles plus one, two, or minus one or two. The method is whole class drill with work written on the board and children putting up their hands when they have a problem figured out. Later, combinations that make ten are studied in the context of two-place addition. Subtraction worked in as a natural variation on the same facts. The idea is to work for speed and confidence. When such work was continued in a grade for a whole year, a significant increase in standardized test scores could be detected. However, the major gain mathematically seemed to have been a development of the habit of looking for patterns or relations that could be used instead of just applying a standard procedure to each and every problem. The question, "What if you don't find a pattern?" arose, but children were always able to find a pattern if several of them worked on the same problem. So faith in patterns by teachers, students

and the RP increased, and some teachers began to use this kind of drill too.

4. Teachers can help develop materials written to try to capture some of the problems of primary math (e.g., article on problem solving or case studies of individual children) and this is followed by activities designed to improve the materials for publication in some accessable form. For example, RP's may go over materials with teachers identifying what is helpful and what is not, perhaps collecting new material from teachers and writing it up for publication. (See Brandau and Easley, 1980, and Easley, 1979; Stake and Easley, 1979; Easley and Grieb, 1979).

5. RP may observe (with video or audio recording) a mathematics lesson and present teacher with transcript and/or recording to discuss how the lesson went, why certain things were done, what realistic options the teacher had in the situation, etc.

A teacher, talking with a resource person after school about her emphasis on teaching the process for each type or math problem, complained that her son in junior high school brought home difficult math homework which she couldn't help him with without first getting her husband to show her how to do them. (Her husband couldn't help her son, because he approached mathematics intuitively and didn't believe in process teaching.) This discussion revealed that she was beginning to feel concerned that her competence as a process teacher, in which she had taken great pride, was developing in her pupils the same kind of dependence on being shown how that her son had, and that she had developed herself when she was in school. She didn't like being so dependent on being shown the process for every new problem and wanted to develop an approach to math teaching that would strengthen children's confidence in

their own intuitions. However, she did not want to try the suggestion of giving her pupils major projects like measuring the size of the classroom, estimating the number of bricks in the school building, measuring each other's height and the distance they could stretch between their finger tips and then making graphs of the relationships they found. A project approach would be too chaotic for her. So attention was directed to finding a way to develop pupil intuitions that was more comfortable.

6. An RP may serve as an aide in a math class, working under the teacher's direction and discussing the lesson and particular problems of individual children afterwards. This is one of the better ways for RP's to learn what teacher's problems are and what are and are not realistic approaches to solving them.

A teacher has been working with one resource person on a weekly basis for over a year, and with two other RP's on a shorter basis. She has evolved an understanding and appreciation of the fact that teachers and resource persons think about teaching arithmetic in quite different ways, and the differences are natural and acceptable. This teacher wrote, "We have developed a relationship where each values the other's opinions, where genuine caring for helping the other has been established, where there is time during the week to mull over and reflect on ideas presented in our afternoon sessions, and where my ideas and thoughts are just as valuable as theirs." One of the resource persons could have written this just as well, but it happened that the teacher felt free to express herself this way. The RP's are learning how teaching arithmetic can be more like teaching reading because, the teacher is trying to make it so. The RP's are trying to help her do it, but might never have thought of the analogy without her help. The RP's are

learning how a teacher thinks about her job and how to be helpful to her as she thinks about it, as well as while she does it.

7. Clinical interviews with children to discover the reasons for various difficulties have been conducted mainly by resource persons but by teachers and principal also. Video tapes of these interviews are available for loan or copying, as well as case studies and other research studies based on them. Several opportunities are opened up for teacher involvement:

(a) Teachers can identify problems for RP's to study.
(b) Teachers can practice interview technique with RP and do own investigations.
(c) Teachers can study existing tapes and make selections for editing tapes for use in inservice workshops.

8. A day-long conference between teachers and RP's, alternating between small group sessions looking at videotapes of classroom or interview situations and large group discussions. Grade level meetings with teachers from several schools permits discussion of key problems of the grade. RP's have the difficult challenge of selecting tapes or other presentations that will be stimulating and interesting to most teachers. (Car pool seminars could be helpful.)

Ideas which have been proposed but not yet tried out include these:

1. Teachers (2 or 3) from one or two schools go for a visitation - conference to another school where they visit classes one day (Friday) and talk with the teachers they observed individually or in groups the next day. This permits the teachers visiting to follow up on new ideas they have picked up from observation and the teachers who were observed to get reactions from other teachers to the lesson they have taught. While travelling, a "car pool seminar" could also be helpful.

2. Attend "miniconvention" of NCTM in Peoria, on March 27-29, 1980, to listen to presentations of specialists and participate in talk workshops on numeration, basic facts, subtraction, division, etc. Also, the group can meet with several RP's to evaluate what they are getting out of NCTM program. (Car pool discussions on the way there and back again could be interesting.

Doubtless there are many other ways in which the RP's can learn to be more helpful to teachers.

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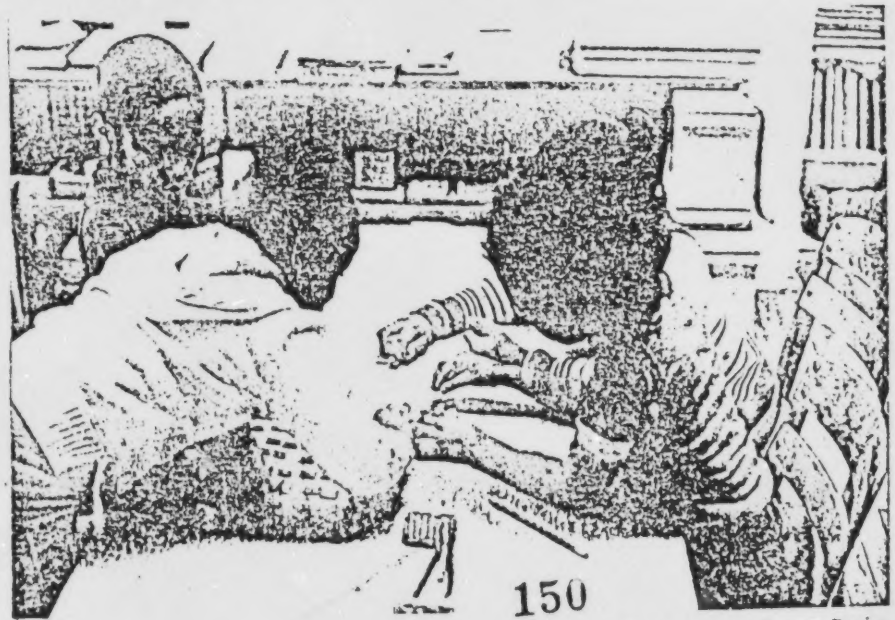

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photos by Ivan Ruiz

An Idea

Aimee Grieb

The change in modes of learning when a child enters school

It makes sense to consider that before entering school, what the child has learned has primarily been a spontaneous reaction to the pre-existing order and structure of the environment surrounding him. There is no idea that, "I must learn to speak in order to earn a living some day", or "I must begin to understand the concept of weight or my parents shall punish me", etc. Although children are taught certain rules, songs or how to count, the most substantial part of their learning seems to happen without any apparent ulterior or extrinsic motive.

A substantial change occurs then when the child enters school, they are now being asked or required to learn specific topics designated by the teacher, e.g., addition and subtraction. Suppose the child does not care about addition or subraction. The teacher then can provoke the student to care about learning through such behavior as giving preferential treatment to those who do learn, e.g. praise, respect, appreciation and by denying this treatment to those who don't learn. In other words, to say 3+4=7 is the good and valuable thing to say rather than 3+4=8.

I am not condemning teachers for how they are motivating their pupils to learn, and I fully believe that everyone should know by heart their basic addition and subtraction facts. What I am critizing, or better said, wanting to draw an awareness of, is the new mode of learning school requires of its pupils. Rather than spontaneous, unaware learning, learning which is not outrightly selective, the child is now being asked to consciously apply himself to the learning and understanding of a huge amount of specific material.

Note the fundamental skills a child is asked to learn in first grade: reading, writing, and arithmetic plus other selected skills or topics. In other words, now the child is asked to actively seek to acquire a specific set of skills, knowledge and understanding, he is asked to direct his attention and thought and time to this acquisition. This is a very different manner of learning than the child has experienced using thus far. The child must now start learning how to learn, how to consciously direct his activities to understand, to appreciate or to know a topic or skill. Whether he is successful in his learning depends upon standards established by the teacher with respect to what behavior indicates the acquisition of these topics or skills she is presenting.

In essence then, a teacher has two tasks:

1. to impart the material that curriculum experts, school systems, and they themselves decide is valuable; and

2. to help the student learn how to educate himself consciously and selectively.

What is next most important, is that, if one believes that learning is ultimately the activity of the student, then "1" cannot occur without "2." Teachers are told to do this, to teach that, but the method for stimulating the child to learn this and that is not so readily available and is the thing that teachers learn through teaching itself. I believe this is what teachers are saying when they say that college didn't teach them how to teach, but they learned through doing. They learn by facing the situation day by day, by trial and error. They know precisely what they want to teach, but they do not know how to teach until they have a number of years of experience, if ever.

Jamming with Third Graders

Aimee Grieb

Jack has just finished giving a series of oral problems involving "mental arithmetic." He had been giving problems with the purpose of establishing the correlation between adding ones and adding tens, for example: 1+2+3+4+5 = ? followed by 10+20+30+40+50 = ?. I began by giving a similar problem, although quite a bit easier. Through observing Hal Taylor teach, I have picked up on the idea of beginning my conversations with a classroom with problems that I believe every student will know the answer to.

Alright, I've got one for you. 10+10-10+10+10-10. How many people got 10?

No one raises their hand, so I ask about the next highest ten.

How many people get 20?

A number of children raise their hand, although not all of them. Thus, I ask for the next highest ten.

How many people got 30?

In asking how many got 10, 20, or 30, I am trying not to give the answer away. I want them to be able to establish for themselves what the accurate solution is, and to feel secure about their solu-

tion without requiring my approval. Essentially, I want them to be able to derive a method for reaching solutions that they feel is accurate, and by which they have a good amount of confidence as to its correctness. Unfortunately, while asking these questions, I had lost track of what my original question had been. Something in the back of my mind told me that I had neglected to keep track of the number of 10's that I had added or taken away. In other words, I was thinking that I should have been aware that I gave out two 10's, took one 10 away, gave out two more 10's, and took one 10 away, thus leaving me with two 10's. Noticing what I could have done in order to make the problem easier to organize and deal with became the focus of the remainder of my lesson. I had not planned this to be the focus while watching Jack teach, the idea arose when I recognized its usefulness in solving the first problem I gave. Although I have never participated in a jam session with jazz musicians, I have done a lot of improvisation with movement, in dance classes and in performances, and this lesson is in essence of the same nature.

I've lost track in the middle of that one. I was going to go by what was most popular. Okay, here we go. This one's going to be hard; I don't want to make it too easy. 50+50+50+50-50-50-50+50. One more time? (Some say "yes", and some say "no.")

I am now using my fingers to count the 50's as I gave them out and take them away, with the intention that this will help communicate the more efficient way for doing these problems. I want

them to keep track of the number of 50's, or sets of 50, that are being presented. This is why I will be asking for solutions in terms of sets of 50 or as multiplication problems.

Okay, those of you that have it in your head, you keep it there and...

(Some of the children have the solution, some do not. I want those with the solution to be recognized and at the same time am waiting for the other children to find it.)

I'm gonna go over this just one more time. 50+50+50+50-50-50+50. (A lot of 000's accompanied by raised hands from the class.)

Okay now, do I have 50? (No one in the class agrees with this.)

Is my answer 2 times 50? (Almost everyone says "no", but one person says "yes".)

Hearing that someone thinks the answer is 2X50, I start to explore what 2X50 means.

What is 2 times 50? I wonder what two 50's are. (A substantial number of students say it is 100.)

Okay, is my answer 3 sets of 50? Do I have three 50's in my answer? (Some say "yes" and some say "no".)

Once again, because of the disagreement, I explore what it means if I have three sets of 50. I am assuming that the difficulty is coming not from the problem itself, but from the manner in which I am expressing the answer in sets of 50.

How much do I have if I have three sets of 50. I have...(I am now writing the numbers on the chalkboard)...50+50+50. How much is this altogether? (They are saying different answers.)

I realize that they are still thinking about what the answer is, so I give them more time to think.

What is it? (More students are now responding 150.)

Who got 150 for their answer? (A lot of raised hands.)

Thank you.

I notice now that the 150 is not a response to the initial problem itself, but is a response to the discussion about 3 sets of 50. I was not sure that they understood what I meant when I used the term "sets of" as a replacement for "times." I know they are acquainted with multiplication, but I do not believe they are aware of the idea of it being seen as a certain number of sets of the same quantity.

Okay, let's try another one. This time I'm gonna work with the number 20. Okay, so let's get the 20's clicking. Alright: 20 plus 20 plus two more 20's... (a lot of Oh's and raised hands from the students)... (pause)...and take away 20. (pause for them to think) 20 plus 20 plus two 20's take away one 20. How many 20's do I have left? (Most of them answer "60.")

This is not what I wanted to hear. I am still trying to get them to count sets of 20 and apparently they are still adding and subtracting so I give the numbers, which is what they were doing when Jack was teaching. I was not aware at this time that I was asking for a substantial change in their manner of computing the problem. I do persist though in trying to get them to think about sets of 20 rather than total amounts.

Okay, I've got 60. (Pause) How many 20's are there in 60? Is there one 20 in 60? (Most of them immediately say no to the idea of one 20 in 60 and yell out "three.") Three 20's? Okay, three sets of 20. Alrighty.

Okay, this time I'm gonna work with the 30's.

The class is remarkably enthusiastic at this point and are very anxious for more problems to solve. I am responding to this as I usually do by getting a little dramatic. I am now acting out the part of an auctioneer.

Okay, this time I'm gonna work with the 30's. We're getting bigger. Okay, I've got 30 once, I've got 30 twice, I've got three 30's, I've got four 30's. What time is it now? It's almost 2:30...

I am playing with the double meaning of "thirty," that of being the quantity 30 and that which is applied to time specifically. I believe it is my attempt to draw their awareness that 30 is not a number isolated in mathematics, but that it is a quantity that they often encounter in telling time. It is a bit of a pun, mainly to entertain myself, and perhaps them.

To emphasize again that I am dealing in sets of a specified quantity, I repeat the problem as follows:

...I've got four 30's and I take away two of them. How much do I have? (One girl says two, most of the class says 60.)

I realize now that by asking "how <u>much</u> do I have?", I am asking for a total amount in which 60 would have been the appropriate response. I should have asked, "How <u>many</u> do I have now?", to elicit the idea of having two 30's.

Because most of the class said 60, I go more directly to drawing connections between sets of a specified quantity and the total amount that it represents.

Okay, I have one 30 then I have two 30's - how much is two 30's? (Everyone yells 60.)

Now I've got three 30's - how much is that? (Everyone yells out 90.)

I recognize now in retrospect, that I was going against my intention of the lesson by having asked for these equivalencies as I do. I was promoting their thinking in total amounts rather than thinking in sets of 30.

Then I've got four 30's. I'm getting big - how much is that? (There are no answers coming from the class except a lot of mumbling. No one seems sure about what it is.)

Recognizing that they are now over the line between what they can immediately respond to and what they think about, I go to the chalkboard and write out four 30's in a column.

Okay, 30, 30, 30, 30. (written on the chalkboard in a column)

(I circle the top two 30's.) Okay, two 30's is how much again? (Everyone yells out 60.)

So two 30's is 60. What is 60 plus 60? (I circle the bottom two 30's in the column to represent the second 60 of the sum "60 plus 60.")

(Not many students are sure of what the answer is, but some are yelling out just 12 then 112.)

I hear them respond "112" and I am aware that they are getting two concepts mixed up; the concepts are not yet coordinated with one another. The first is that I know that they know that 6+6 is 12. The second is that because they are dealing with not 6+6 but with 60+60, they recognize that there is a place value change which is going to take the total into the hundreds. The answer comes out then, "one hundred and twelve." I really have no idea how to clear this up, but I am wondering at the time if they are seeing 120 on their minds and misreading it because of the just mentioned conflict, or if they actually believe it's 112.

Is it this number...(I write 112 on the chalkboard and hear many yeah's)... or is it this number...(I write 120 on the chalkboard and again hear a lot of yeah's.)

Now things are really in a mess. They are not at all sure of what 60+60 is because of this confusion with 6+6 being 12 and changing place value. They are taking shots at different answers like "one hundred" or "ts higher, much higher," and I have really no idea what to do to clear it up. I am thus deciding whether to just let is pass and do more preliminary work in order to rebuild to this point, or if I should now work with them on 60 plus 60 precisely. For some reason or another I decide that we need to clear up 60+60. If I had been in a different frame-of-mind that day, I would have chosen the former instead.

Okay, here now, wait a minute, now listen to this little discussion about how to write 60 plus 60, cause this is important to me. Okay so here we go. (I go to the chalkboard and write 60 plus 60.) We've got 60 plus 60 and I can't decide if it's 112 or if it's this number. (I point to the 120 previously written.) Now I don't know which one it is. (Pause) Anyone got any ideas?

Is it this one or is it this one? (I point out 112 and 120.)

(There is a lot of talking in the classroom which initially is centered around 112 but seems to be moving to 120.)

> I now choose to deal with 60 plus 60 as addition in columns. I do this because I do not see any way to directly talk about 60 plus 60 as being 6+6, except moved over one column. I therefore resort to using the method of addition that I know they have been taught. I dislike very much resorting to the use of the addition algorithm, but when I found myself in the middle of this confusion about 60 plus 60, I became too concerned about getting them to say 120. In other words, I was end-gaining.

What is 0 plus 0? (Everyone answers zero, so I write that in: 60+60=0.)

What is 6 plus 6? (Everyone answers 12 so I write that in: 60+60=120.)

(At this point they start to cheer and applaud when they see 120.)

This class is amazing in that they can get so excited about seeing the problem solved.

Okay now, who can tell me what this number is? Who can read this number? (The number below 60+60=.)

(Some people respond 120, some still say 112.)

Is this 120...(some kids say yeah)...or is this 112? (Most say 120.)

How do I write 112? (They respond with one-one-two.)

Okay, okay. Now we're back in business.

I am noticeably relieved that the distinction between 120 and 112 has finally been accomplished, although I wonder now if they would say 140 or 114 if I asked for 70+70.

So I've got 30 plus 30 plus another 30 plus that fourth 30, and you all who know it and keep it in your heads, and now I take away 30, how much do I have?

("120", "40" - different answers coming out, so I go to the chalkboard and write as I speak.) 30, 30, 30, 30, and I take away one of them. How much do I have?

(Most students say 90. I ask one of them specifically.)

How much do I have?

(He answers 90.)

90, right. Okay, let's go on to the 60's. (There are a lot of ooh's because this is a big number.) Okay, I've got one 60, I've got two 60's, I've got three 60's. How much altogether do I have if I have three 60's? (Most of raised hands and ooh's from the class - a number of them know what the answer is.)

At this time the class is not yelling out the answers and I believe the reason they are politely raising their hands and waiting to be called on is because I am now much more relaxed having finished the discussion of 60 plus 60. Their response to my feeling again back in control and in good order is to give me the same. That is, they respond by giving me the control and good order in their behavior. This is a fascinating phenomena.

Not enough students know the answer yet so I go through the problem piece by piece, also to emphasize our dealing in sets of this specified quantity.

If I have one 60, how much do I have? (Most students raise their hands anxious to give the answer, so I ask one student specifically., How much is just one 60? (The student answers "60.") And if I have

two 60's, how much do I have? (One student blurts out 120, but I ask another student specifically, and he answers "120.")

Now, if I add one more to that group, how much do I have? (A lot of raised hands and ooh's. Someone yells out "180.") Okay, let's keep it silent until we see what everyone has. (Pause) How much do I have? (I ask this of one student.) How much do you think I have? (She responds, "180.")

(I ask the class as a whole:) Does everyone agree with that? (They all say "yeah.")

I agree too, I think that's the way it is.

I have purposely said this last statement to indicate that the math we use in arbitrary, and that according to this system, 180 is the same as three 60's. I try to separate out that the correct answer is not decided by and/or in me as the teacher but that the correct answer exists according to the principles and organization of the system of mathematics being used. Me as the teacher and they as the students are essentially in the same position of needing to apply our intelligence to this sytem in order to recognize its nature and manner of functioning. I am aware that because I have more experience, the answers come more quickly and easily to me, but the students do not yet see this and misinterpret to believe that I have some different and all-authoritative means for determining the answer to the problems I give them. I dislike this

creation of a notion that a teacher thinks differently than her students and that she is the final determiner of what is right and wrong, so I drop in there lines like "I think that's the way it is too" to undermine such notions.

Because I thought using 70 would be too large to handle, I decided to use 7 instead in what follows.

Okay, let's try something. Let's try some 7's. One 7, two 7's, three 7's, four 7's, take away 1. (I repeat the problem and let them think.)

Have I got 28? ("No!")

Oh, I must have 29 then. ("No!")

Do I have 26? ("No!")

I know what I've got, I've got three sets of 9. (No response.)

No one is answering because they don't know what to make of this thing of bringing sets of 9 into the picture when the problem has been given in sets of 7.

Do you all agree that I have 27 (they immediately start yelling "yes" when they hear the answer they have been thinking come out, but I had not finished the sentence yet) if I have four sets of 7?

What I was thinking of asking was, "if I have three sets of 9." The class was too preoccupied with hearing the "27" though and did not notice my mistake.

How much is four sets of 7? ("28!")

What if I take away 1, what do I have then? ("27!")

Okay, thank you. Okay, one more now. Alright, I have two 5's. I add 5 to that, two 5's and I add 5, and then I take away 3. Everyone got it in their heads? (Some answer "yes"; some answer "no.")

Do I have 3 times 4? (One girl answers "yes", the rest say "no.")

Here is the point where one student recognizes very quickly that her answer of 12 is the same thing as 3X4, but she is the only one. Everyone else disagrees that the answer is 3X4, they say the answer is 12. She is very confident, in spite of being the only one to respond, "yes." I look at her to indicate my agreement with her, but I ask more questions from the rest of the class in order that they can recognize as she has already recognized.

Well, let's see. How much do I have? 11, 12, 13? (A number of people answer "12.")

12 (I give my affirmation to this response.) And what is 3 times 4? (They answer 12.)

I close my piece of this lesson, hoping that I have left them with a concept that will become apparent to them later, that being reflectivity, i.e., if a=b then b=a.

On the Difficulty of Communicating What I've Just Learned About Teaching Place Value

Jack Easley

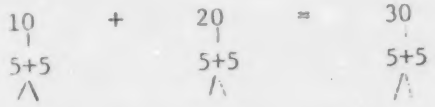
Primary grade teachers (and others too) have brought problems from their experience to the attention of mathematics educators and psychologists in the form of statements like this, "I don't know why, but whenever I teach some particular topic, there's somebody who already knows it and several students appear beyond all hope of ever getting it." If we choose to interpret such complaints as challenges of educational theory instead of as reflecting the naivete of teachers, we might discover that there are critical weaknesses in the dominant learning theories of the day. It may be our cultural norms in the community of experts that reject seeing these as expressing dissatisfaction with the key metaphors of learning theory on the basis of personal experience in teaching. Of course, learning theorists can always account for such common-place occurrences in terms of whatever theory they have at hand. Perhaps the teachers had hoped for a simple, direct way of understanding teaching and learning, and they don't find the simple ideas they were taught at universities satisfactory as approaches to the phenomenon. Having practitioners challenge fundamental paradigms of psychology is reminiscent of the comment of some observers that children often challenge the paradigms of the scientific disciplines. Some philosophers are fascinated by this (e.g., Piaget, Hawkins).

Terry Denny reported (1978) that teachers spoke to him of "the great mystery" of forgetting and an equally great one of sudden unexplained clarity. For example, one of them said:

> Except for the "block 1s" (35 of 140 third graders) all third graders know one thing perfect in math one day and the next it is gone . . . I mean GONE. (She has all the block 2s -- average kids.) It is a week to week, year to year experience. I have had children for three years in a row. Knows his facts up to 19 in addition beautifully in May, comes back in September and he's forgotten them. I mean zero retention. Now what I want you to know is that ALL the kids except block 1, and some of them too, have this mysterious "forgetting disease." Last week I had the "2s" borrowing and carrying and I was so happy and we were all dancing. After Thanksgiving they came back and acted like they had never heard the word "borrow." Borrow!? Teachers live with this. They see it every day and get used to it, so we don't talk about it. But it's there teaching math; day to day.
>
> I taught the basic addition facts in the first grade to these very same children. I have complete confidence in the (mathematics) teaching of my friends here in the second and third grades. I know they did it. And we use all available manipulative materials, counters, popsicle sticks, filmstrips, flashcards, you know. Now I have the same little people I had three years ago. The mathematical understanding is still there, but the proficiency is not. Summer (vacation) forgetting hits them every year. I can tell you they had it to 18 (the basic addition, subtraction facts, no carrying, borrowing). They lose it, find it, lose it. Maybe none of the 18 is really a part of your system, a part of you. It's too-much-too-early maybe. Number readings is [sic] greatly overemphasized in kindergarten. We take pride in teaching more, faster, earlier. Teachers (first grade) will say, "I had my top group in second grade math in February." We really don't know what difference that makes.

Steinberg and Anderson (1973) present a case study called, "Teaching Tens to Timmy", in which after painstakingly trying all the methods they could find, for getting Timmy to grasp that 24, for example, meant two tens and four units, inexplicably one day, Timmy began to respond perfectly to such questions as though he had known about tens all his life. I had worked on that case and with Bernadine Stake's (1980) analysis of the conflict between counting and place value, which teachers had reported to us, and I had worked on several other parallel cases in science and mathematics education [e.g., dissertations by Driver (1973) and Erlwanger (1973)]. This same problem of "tens and ones" puzzled me deeply for a whole day recently, while I was trying to help four primary grade teachers, three of whom were concerned with children who were either having trouble with the introduction to tens and ones or were having trouble with the use of tens and ones (and hundreds

too, of course) in the algorithms I think of as borrowing and carrying. I found myself repeating all the devices I could think of and trying them with what were generally very willing and eager children. The trouble was that, at the end of each of these little lessons, the original problem remained. I even invented a couple of different versions of a stick figure representation in which the 1, 2, 3, etc., placed in front of a stick-person's head, implied 10, 20, or 30, etc. fingers.



By this device I hoped to bridge the gap between object representation and base ten numerals. But, in the end, I knew it was hopeless. I couldn't replace the figure's head with a numeral for units and get away with it.

However, a couple of hours later, it suddenly dawned on me that, inexplicably, I now really understood the whole puzzle. The pieces fell into place so perfectly I could not easily get them apart again to describe what each of them was. Here was a whole new picture which I alone understood -- or so it seemed to me -- but then I recalled that dozens of researchers and thousands upon thousands of teachers must have struggled with this problem, much as I had, and some of them must have also had the pieces fall together into a beautifully clear picture, which they could not easily communicate to anyone else. Probably, many of them had "pictures" very similar to mine. One problem this event raises for me is to find out why the pieces of my puzzle stayed apart for so many years of active research on children's conceptions of number. I would like to help others examine the pieces in their own minds that won't go together right. Meanwhile, for the record, I'm going

to put down what I can of this insight, describing the pieces as I originally knew them and as I now know them in the picture I have at the moment.

This is the situation after "the moment of insight" in many occurrences of what Brandau and Easley (1980) call Problem Solving 2, solving a problem for which there was no ready-made method of solution that could be selected from a small number of available alternatives (that's called Problem Solving 1). Like the teachers described in that paper, I have solved a pedagogical problem (at least to my immediate satisfaction), not a mathematical problem. It remains to be seen whether it helps resolve a problem of psychological theory. But whoever reads this will likely be now nearly tearing their hair, because I have not yet even tried to communicate the solution I claim to have found. One reason I'm being a bit slow about it is that I know that most readers will be inclined to say, "Oh, that's obvious, I don't see why it was so difficult for you" or they'll remain extremely skeptical about my new insight, feeling, from past experience, that these moments of "whole truth" are not to be trusted because the beautiful picture they produce tends to crumble away. I also know that, when I get back to the reality of school and children again there will be many obstacles to the use of my beautiful picture, some of which I can't even guess now. I am delaying presenting the solution, also, in order to call attention to the emotions playing in me. I'm like children discovering the third ten in $\frac{15}{+15}$, excited.

Emotions are to be expected in Problem Solving 2. Also, communication of solutions may be very difficult, and self-evaluation and criticism is likely to appear prematurely before the difficult task can be accomplished of taking the new picture apart and demonstrating the equivalence of each of its parts to well-established bits of knowledge. To satisfy the impatiently curious who can't wait to get the end of such a timid writer's story, I present a brief synopsis of my new picture in the next few pages, with the full

realization that many of my readers will not believe it, but with the hope that some will be motivated to hear me out. Then, I will pick up the struggle of tearing out pieces of the picture and showing how decades of hard thinking research can be understood as strongly supporting each of them. That rationalization will be addressed to the research community. Then I'll still have the even more difficult job of finding what it is that most teachers already know that will enable them to construct a useful picture of their own that obviates this particular problem becoming a nuisance in their work. The first of these two tasks I've been trained to do. My career has been that of a theoretical analyst and clinical researcher, in the Piagetian sense. I know quite a bit of the literature. What I don't know is why I'm having better insights helping teachers than I was taking their pupils out of class to interview.

My second task has no supporting discipline. It is an in-service teacher education task related to a very specific content area. I took the title of Professor of Teacher Education, realizing that there was no such discipline, because there are some very important problems to be solved. Perhaps if a large number of such particular content specific problems get solved, analyzed, and ways are subsequently developed of making the solutions of their problems so clear to teachers that they can operate in the classroom with a clear mind, then we may have the beginnings of a discipline of primary mathematics teacher education. (Perhaps I should get my title changed to read, "Exploratory Professor of Teacher Education." That would license me to continue to try out new ways of helping teachers solve their problems in a more and more disciplined way.)

The Problems of "Tens and Ones"

There has been a long concern of teachers and mathematics educators to find a better way to help with the "place value" problem. It has been satirized in song by mathematician Tom Leherer in his song about the subtraction algorithm. Dora Whittaker puzzled me and a few primary teachers, several years ago at Steuben School, by recommending there be no counting in school beyond nine without explicit recognition that next, one begins to count tens and ones. She recommended that sticks of ten Unifix cubes should be assembled and counted to train children in the convention of writing "one ten" and "no ones", etc. Kindergarten was the place to start, and "spelling" approaches to writing numerals beyond 9 were to be avoided at all costs. I suppose most of us hadn't even grasped the problem that she had already solved, how to avoid having a conflict between counting and place value. Realistically, I thought, the spelling approach to writing numbers of more than one place is too well entrenched in home and school to be eradicated. The problem I was more concerned with is what to do with children, and how to help teachers, who are caught in the midst of re-grouping, re-naming, borrowing, carrying, or whatever the algorithmic role of the base-ten, place value system happens to be called. But I needed to understand that problem at a more fundamental level than I had to date.

The symptoms of trouble may appear before any occasion for formal algorithms like those appear. Many children in first, second, and third, grades cannot immediately give the answers to problems like: $10 + 4 = ?$, $40 + 9 = ?$, or $22 + 22 = ?$, because they conceptualize them in terms of the counting or "counting on" process they have been taught for adding pairs of numbers less than ten, rather than thinking of them in terms of representations of so many

tens and so many ones. From that point of view, these problems are trivial, but as counting problems there are many possibilities in them for error, boredom and frustration. When such children are asked a question like, "How many tens, and how many ones?" it either seems to lead them to confusion or to rote answers from which nothing follows logically. While we want rote answers to 2 + 2, 3 + 3, 4 + 4, etc, and to "What comes after 4?", "What comes after 5?", "what come before 9?", etc., we need inferences from those facts and the tens and ones convention for constructing numerals to what the total (or similarly, what the difference) will be. (I believe that arithmetic, like mathematics in general, can be best presented as a few conventional rules, a few definitions, and a few essential facts, and the larger and more exciting part of it, as consisting in creative and exploratory approaches to solving interesting problems.)

The pupil's major hang-ups in making inferences, as borne out in dozens of studies of pupils of different ages (Driver and Easley, 1978) seem to be due to their prior conceptions of the subject that are disregarded in teaching. That is, the pupils do make inferences, but ones which give wrong answers most or, at least, some of the time. The resulting negative "feedback", even if kindly given, tends to make children insecure with their inferences. The joyous shout of "eureka" does not sound too often in most primary mathematics classrooms. The task of the teacher who wants to help children with inferences about tens and ones, and the resource person who wants to help teachers help children with such inferences need to find out the prior conceptualization that is causing the most trouble. In most cases of "place value" conflict, thinking of counting as establishing a one-one correspondence between objects and number tags (Gelman and Galistell, 1978) is the conception that stands in the way of grouping objects and counting each group of like size. That's the key to multiplication and the tens and hundreds columns in place value.

Stake (1980) examined separately the conflict between counting and place value and between counting and multiplication. What I have just realized, from working in Stueben School with Aimee Grieb on February 1, 1980, is that place value is conceptually a form of multiplication in the sense that Stake discusses multiplication, that is, counting groups of identical size. Counting the groups, as she points out, is not the same as counting units by twos, fives, tens, hundreds, although that process, called "skip counting" may be useful, in conjunction with counting the groups, to obtain the answer to a multiplication problem. The difficulty is that, conceptually, counting groups of tens and counting by tens are quite different, but both are essential and must be integrated, if confusion is not to result.

	Tens		Units	
	1	○○○○○ ○○○○○	(10)	
counting groups of tens	2	○○○○○ ○○○○○	(20)	counting by tens
	3	○○○○○ ○○○○○	(30)	
	4	○○○○○ ○○○○○	(40)	
	⋮		⋮	

To know that three tens is written by placing a three in the tens column and that, if there are no other quantities added on, the total (product) is thirty (and other facts of this kind) is to know the tens multiplication table. We expect children, however, to begin using this base ten numeration system before they have been introduced to multiplication at all, even in the sense of counting groups of objects of the same size. It seems that children struggling with the place value algorithm problems alluded to earlier are in need of learning to count groups of tens by ones and to correlate that counting with counting them by tens. Both of these systems of counting, however, violate the one-one correspondence principle that Gelman and Galistell (1978)

demonstrate is built into children's counting from the age of two or three. Both systems require one tag for each group of ten objects (one-ten) but they employ different sequences of number tags. To grasp that one of these gives the product, requires thinking of the whole collection of groups as though they had been counted individually with the standard set of tokens. Why should counting by tens give the same answer? It is probably easier to learn this by forming relatively solid groups (e.g., sticks of ten Unifix cubes each) so each physically becomes a single object, yet can obviously still become ten unit cubes again than to understand what it means to trade-in ten unit cubes for one ten stick. By six-year-old transductive logic (Piaget, 1930) counting groups of tens by ones is a violation of the following rule: When you count by tens, you must point to a group of ten objects each time you say one of the tags, 10, 20, 30, etc. From this, children can infer transductively that when you count by ones you must point to only one object each time you say one of the tags. Perhaps, easiest of all, according to Hal Taylor, is trading with base three blocks, where counting isn't needed at all. This is the most persuasive argument I've ever heard for multi-base blocks.

Hal Taylor, two weeks later, told me that he figured out that the difficult problem of telling time involves the same double counting problem that tens and multiplication do and that counting by fives or tens don't. Then it occurred to me that another problem teachers have been asking for help with, "making change"--counting money or putting out the coins for a certain amount of money. I recently looked into a Japanese first grade textbook to see how they deal with the business of tens and ones and found some suggestion of sensitivity to the whole collection of problems, telling time, and coins included.

Lesson 1: Eight pages of pictures of objects and children that can be counted, with no more than ten in any one picture.

Lesson 2: The numerals from zero through ten, with arrows showing how to draw them, and objects to count to go with them.

Lesson 3: Two pages on ordinal and cardinal uses of numerals.

Lesson 4: Combinations of sticks and circles that make 5, 7, and 10 (two pages).

Lesson 5: Two pages of plane and solid geometrical shapes.

Lesson 6: Six pages of tens and ones and writing the teens, mainly using bundles of ten but also bringing in a number line.

Lesson 7: Six pages on adding single digit numbers with sums up to ten.

Lesson 8: Subtracting from numbers below eleven (six pages).

Lesson 9: Two pages of clock faces with pictured activities (half-hour and on the hour only) and four pages of review of arithmetic.

Lesson 10: Four pages of word problems adding and subtracting with pictures and number sentences. (This is half-way through the book for first grade.)

What seems interesting here is that no counting, reading or writing numerals beyond ten occurs until tens and ones have been introduced. Then adding and subtracting below eleven is worked on, presumably to avoid using counting up into the teens as an algorithm for adding and subtracting. It is also interesting to note that Robert Wirtz' New Beginnings (1980) suppresses counting in a pre-writing experience in number patterns including tens and ones.

Why Counting Algorithms Are Stressed

Counting algorithms were notorious in my own school days, and presumably in those of most adults over thirty, for counting on one's fingers was a school tabu. Presumably, most of us pupils did it with varying amounts of concealment and guilt. The school goal was instant recall of the facts of addition and subtraction--and later, multiplication. The use of oral drill on facts, with or without flash cards, was traditionally a unison practice, and only individualized in special cases.

The feelings of primary-grade teachers about counting and oral drill seem to have reversed rather strongly in the last ten years or so. Some attribute it to the "process" emphasis of the "new math". K-1 teachers now often teach a counting algorithm for each problem type: addition, subtraction, missing addend, missing subtrahend, deciding which number is larger, etc. Often silent chips, counting sticks, beads on a wire, or other manipulable objects are used and children are "weaned" from them to making marks on paper, but there is still a frustrated hope that quick recall of the facts will result. The children often persist in frequent use of counting algorithms up to the 4th grade. It is probably irrelevant what was intended by the "new math" promoters. The plain fact is that drill is a better method of memorization than counting algorithms because knowing an algorithm obviates the need to recall facts. (Psychological learning studies support this.) The defense of "Process teaching" is that counting algorithms are meaningful and drill is not. On the other hand, Taylor presents a combination of drill on doubles and the decompositions of 10 with more efficient algorithms one or two more and one or two less, that preserve the meaning of structure of the natural numbers.

As the square shown below makes clear, once the combinations represented by the margins and the two diagonals are known, every square can be reached by adding or subtracting one or two, i.e., starting with either a double, a decomposition of 10, or the decoding of teens into tens and ones, the rest of the standard 121 addition facts are generated very quickly by the "one more or less" or the "two more or less" algorithms.

0	1	2	3	4	5	6	7	8	9	10
1	2								10	11
2		4						10		12
3			6				10			13
4				8		10				14
5					10					15
6				10		12				16
7			10				14			17
8		10						16		18
9	10								18	19
10	11	12	13	14	15	16	17	18	19	20

Can Alternative Conceptions Be Put In Textbooks?

A textbook usually tries to be complete, so anyone who really studied it all could do any problem it presented. Math texts usually fail for most students, in this effort, because teachers (and even tutors) are needed for all but a few of the students, who, remarkably, seem to be able to do many problems without even studying the text and easily catch on to the rest of them. Usually, in a text, only one approach to a given problem type is presented. Even carefully programmed texts often failed to communicate math conceptions accurately (Erlwanger, 1973).

Clinical interviews and other cognitive studies strongly suggest that students construct what they understand from previous things they have understood--whether these be correct or incorrect understandings determines the degree of success. However, the existence of these prior understandings is ignored by textbooks and by most teachers. The task is to identify which understandings are causing which kinds of trouble and find ways to suppress

those understandings in favor of less troublesome ones. Counting is troublesome for many, if not most, primary school children when it comes to place value. It can be replaced.

We can now explain to teachers, I hope, why it is that students who seemed to understand borrowing and carrying one week can be totally out of it the following week--as though they had never heard of it. If, in the meantime counting schema become activated, then they replace the place value schema in their minds and produce a completely different set of responses. Since there is a conceptual conflict between the two sets of schema, they cannot both be active at once, even if there were enough space in the children's consciousness for both sets of ideas at once. Suppressing counting schema should be helpful in reactivating place value schema.

On Becoming a Resource Person

Jack Easley

Having selected primary school math as a target area, and picked three resource persons who share an interest in helping students think through mathematical problems, when we ask teachers to tell us of their problems, naturally we hear them with a certain bias. We do not, for example, interpret the problems they tell us about as requiring training in associating certain computation algorithms (operations) with actions or "with being given 'parts' or 'totals'" (Zweng, 1979) for we think of problem solving as a creative act. However, helping teachers solve their problems also requires a creative act, so we look for ways in which each teacher can increase flexibility while gaining in ways she (or he) identifies as progress. So far, we have not encountered any direct conflict of purpose, because we have talented resource persons who can usually find a way to achieve some recognized progress on both the teacher's goals and ours simultaneously. Also, teachers are not so strongly committed to the moral goals of math teaching that they would put them ahead of achievement on tests and other goals we share. We even have some moral goals ourselves, e.g., avoiding interference of one child with another's work (see Chapter II).

The policies and principles listed below are exemplary of our resource persons' views and of the general character of our project to date. We are actually working in four schools, with a somewhat greater concentration of effort in Steuben School in Kankakee, where the problems seem to be a bit more acute. Interestingly, in all four schools, the problems are almost identical in kind: difficulties in teaching the very able or the very slow, difficulties in various topics for the majority of pupils, e.g., choosing

the right operation for tens and ones, borrowing and carrying, telling time, making change, and word problems. The diagnosis so far seems to be helpful to teachers in all four schools. The acceptance of our policies by administrators is promising but not definitive in every school. Also, in different schools we may discover that there is a different job description of the key person who supports our work. In one, it's a regular classroom teacher, in another, it's a building principal, in another it's a person in central administration, etc. Curriculum specialists are increasingly overburdened as more funds go to teacher salaries and less is left for support. On the other hand, university and college people may be getting more involved in schools, so we are looking there for resource persons.

The new math projects of the past twenty years have apparently left the practitioners in worse shape than they were before; some reasons are on the surface and can be easily described: e.g., they have the added burden of algebraic concepts inserted into the traditional arithmetic curriculum which are treated as additional separate kinds of tasks to perform instead of the integrating concepts they were intended to be. The failure seems to have been due to the prior conceptions of arithmetic teaching as moral training in following directions, in trusting the answer key over reason, because reasons for the answer in the key were not available to teachers or most pupils. The moral socialization to school, expected by most adults in society, is naturally served by teachers in spite of a limited effort of colleges and universities to prepare them to teach math as a rational enterprise. Arithmetic is a natural vehicle in which there are apparently arbitrary procedures and arbitrary sets of problems to which each procedure can apply, and an authoritative source in the text, the teachers' guide. They perceived the message of meaningful learning and reasoning processes as

memorized algorithms instead of memorized facts. The efforts of some of the new curriculum projects to make mathematics in elementary schools a creative, exploratory discipline have failed miserably because school mathematics was pre-empted by the conception of training in obedience as a personal discipline in following directions and remembering when each rule is to be applied, and explicit training on when to multiply, add, subtract, divide was absent. However, Hal Taylor argues that, properly understood, a discovery approach of primary school math need not pre-empt moral training but can assist it.

Other reasons for the failure of the rational approach are more subtle. See for example, the discussion of the conceptual conflict between counting algorithms and place value algorithms (ch.XI). This conflict, apparently recognized for some time by many teachers, and in Japan, has only caught the attention of other mathematics education specialists in the past year or so (Stake, 1980; Wirtz, 1980).

For whatever reasons, most students sooner or later encounter a point at which they cannot make further progress in mathematics as usually taught in school. The grade level at which this happens seems to depend as much on independence of thought and mind as it does on intellectual ability. In one school district, both gifted and remedial math students are given the same diet of "Mindbenders" and other problem solving tasks. Both groups appear to flourish on these materials, as though they are independent minded students who cannot put up with the standard, rule applying curriculum.

Implementation of an explicit set of goals for school curriculum reform is an incredibly chancy operation, as the history of curriculum reforms and the various studies of implementation show. Vagueness is identified by Huberman (1979) as a deliberate strategy on the part of both in innovation and

school personnel who agree to try the innovation. It protects both against failure. He also identifies the major cause of failure in explicit proposals, namely the transformation of the proposal by assimilation to the prior understanding of the teachers and others involved. At most, "implementation for the practitioners involves reducing the discrepancy between prior experience and the (perceived or objective) demands of the innovation." It is this same Piagetian theory of cognitive schema that has been involved in the conception of the present project as an assistance to teachers in working on problems they identify. This is also consistent with Deweyan theory of problem solving. The difficulty, in terms of instructional policy, is that it depends on an act of faith in the theory, and in the practitioners - - that as their pressing problems get resolved, progress is made on larger issues.

There is some outside direction, however. First, the choice of the grade level and subject matter, and resource persons to offer as helpers, rests on an external diagnosis of a critical problem. Children are much too often acquiring the habit in primary grades of getting teachers to show them how to do every kind of problem in school mathematics. This limits the development of their personal confidence and precludes any experience in bringing mathematical thought into their world. One result is that, the U.S.A. is one of the very few highly industrialized nations which does not and cannot require every high school graduate to have studied algebra and geometry. Nearly half the graduates of elementary schools are judged incapable of studying either algebra or geometry.

In dealing with this problem, we depend on sagacity more than policies. Policies support but cannot guarantee success. Webster provides these helps.

Sagacity--Keen, especially in sense perception; quality of being sagacious.

Sagacious--of keen penetration and judgement; discerning and far-sighted in judging men, motives and means.

Two Suggested Policies of In-service Teacher Education

1. Anything a teacher says that suggests a concern in regards to her/his work should be taken seriously. It may not be appropriate to follow up all such remarks immediately, but some recognition should be given. They should not be dismissed, "redefined", answered out of hope that public acknowledgement of it's irrelevance will drive it away.

The principle behind this policy is that each person's world view accounts for most, but not all, of what that person experiences. People feel concern when something happens that doesn't fit their world view. So signs of fear, or concern, especially when there is a strong temptation to define it away, means that there are discrepancies between the world views of at least two parties. Such discrepancies will affect the course of future work and need to be taken seriously. However, the goal is not to eliminate discrepancies but to understand them.

2. Suggestions to teachers should be embedded in the context of their teaching.

The principle behind this policy is that each person assimilates suggestions to systems of conceptual schemes (frameworks) which are active at the time or which become active at the time, because they claim this particular suggestion as relating to their function. However, when the time comes to use the suggestion, it must now be brought to mind if the framework is not already active. The context of the math classroom is often needed to activate a framework that has been displaced because of something that has happened that required other frameworks. The chances are high that the framework that recognized the suggestion in the first place as one it could hold, will be activated again by the same context. Most people are familiar with the experience of thinking of something you need in another part of the house, and

being unable to remember what it was when you get there. The framework of schemas which recognized the need for something have been displaced by others by the time one gets there, and the original context is needed to reactivate the schemas that generated and thus can recall what is needed. Sometimes, by recalling what one was doing, one can recall what was needed, but the context itself is an almost perfect reminder. This is why suggestions need to be made in the context in which they can be used rather than in another context. Simulations of the appropriate context may be obtained by playing videotapes of teachers' classroom scenes and asking the teacher watching to tune in to what they were thinking at the time.

Sagaciously, we could note that elementary arithmetic is not limited to being traversed in a strict linear order, but it is interrelated and interconnected in a structure something like a tree, in which teachers and children can climb around like monkeys, easily leaping from branch to branch. One thing we intend to demonstrate is that teachers who have never had any inclination to depart from the text book can do so comfortably by working with the integrating structures we are developing for them.

Four Principles of Pedagogical Dialogs

1. The principle that children form a bi-modal distribution on achievement tests, if they are given stimulating problems to work on as a whole group. We have only now begun to explore the implications of this principle.

2. The principle of thoughtfulness means that the obvious is usually false, when seen in the light of thoughtful investigation.

3. The principle of working for love. This one blew us over. Promoting independant thought, we wanted children to strive for their own intellectual satisfaction. But love made this possible.

4. The principle of discovery. There is no rule, no procedure, for making one's own discoveries. You just keep encouraging, begging, providing clues that preserve the dignity of the discoverer, and then it happens. Some children discover several things in an hour - others only in a month. But discovery is such great fun, and it builds one's confidence in facing unknowns.

On Belonging to a School

I wondered whether a resource person can become a full-fledged member of the school? That is, can such a person ever be so well initiated into the faculty and staff of the school that they can know what it is to think like a teacher or a principal or another staff member and thus report to the outside world what the emic concepts of the school are? The very concept of a resource person places them outside the school itself, as a channel from the outside, bringing in ideas, materials, and their experience and skills as teaching, formulating, and diagnosing. However, they may gain a great deal of credibility as they either help to solve a teacher's problem or discover something they themselves can't do and, after a struggle, solve that. Merely informing teachers about the alternative ways of teaching that will avoid the difficulties they are experiencing is not helpful to most teachers, who know that before they can use it successfully, they must have any alternative system extremely well internalized. They know they cannot have any immediate hope for relief aside from particular devices that can be incorporated into what they are already doing.

It is a principle of biological evolution that mutations, to survive, must be successful. If we want to build a new rocket or airplane, however, we do not have to add the new ideas onto the old models and have a machine that will fly at each addition. To one who has achieved a new way of teaching and wants to communicate it, he either must rely on charisma for total

conversions, with all the distortions converts make in it, or th... still the very large task--perhaps impossible task--of finding a sequence of small changes so that each change results in an improvement in the previous system in terms of immediate satisfaction. Fortunately, there is another way - put aside one's discovery to help teachers work out their own evolution. This requires that the resource person learn how the classroom scene is understood by the teachers, individually and collectively, at least well enough to be able to suggest innovations that can be assimilated successfully into the teacher's own practice. The reason that even well-rehearsed innovations typically fail is that neither teachers nor innovators can predict enough of the actual events of the classroom to anticipate and prevent prime causes of failure. (Several examples are given in chs.3, 5 and 10.) The goal, then, for the resource person is not to become a teacher or other staff person, but to communicate enough of these roles, as experienced by typical staff persons, to would-be resource persons that they can learn to function as such, successfully, within a reasonable time.

Once teachers recognize the resource persons working with them as learners, open to understanding them, their problems, perhaps struggling with a problem of classroom ettiquette they have already solved themselves with this class, then the initial barriers to communication are lowered and a feeling of collaborative sharing in problem solving can begin. This feeling, of course, has parallels in other domains of work in which persons who have a commitment to share some portions of their life overcome a period of isolation from each other. However, such a break-through does not last forever. It is not that the original barriers are raised again--if that happens, things are going very badly indeed--rather, it is that new barriers are discovered. For example, when I encounter two teachers or the principal and a teacher in the

process of deciding what action to take concerning a child, or standing in a corridor in a most uncongenial attitude, I feel unwelcome.

That is just my perception, of course, and it stems from the fact that I have not participated as a collaborator in such decisions, the administration of punishment or talking with parents about misbehavior of their children of serious proportions. Teachers have not asked for my help with these problems, perhaps to spare me from such an unpleasant part of their responsibility. (They are much more willing to share the decisions about children who are being considered for special help, for example.) However, after the fact, some cases of strong discipline become a matter for general conversation in the teachers lounge, e.g., we're told about a first grade boy who was cut by an older playmate, a third grade boy who brought a knife to school, etc. Perhaps, eventually, even the more unpleasant aspects of a teachers life may be shared. I hope it is possible that, in principle, every pupil in the class could be discussed, sooner or later, in a pedagogical dialog between a teacher and a resource person, if it went on long enough or occurred frequently enough.

Disorder and violence in schools constitute a major problem, a growing tide spreading from inner-city schools into the suburbs and rural areas. Civil rights legislation and Supreme Court decisions that require due process for expelling students get much of the blame. Teachers are the main force for containing school violence with principals, police, truant officers, and various youth organizations assisting where it leaks out of the classroom. Finally, the courts mop up the captured perpetuators of violence.

Teachers, obviously, need all the help they can get. Though their methods may not always appear to the resource persons to be humane, enlightened, or in keeping with the latest psychiatric theory, they are the only major

force dedicated to rescuing children from falling into the [illegible] of a violent life. Math classes are not the only classes where help may be needed, but because, in the words of some reporters (Stake, et al, 1978) they are "dull, dull, dull", assistance here might be of greater value than in some of the areas where the curriculum is usually more interesting. There may be a connection. One inner city high school math teacher said, math is where the one useful thing that children can learn is to "do as they are told."

Understanding the connection between mathematics and a useful social etiquette is very difficult for most resource persons--who, unlike most elementary teachers, think of mathematics as creative reasoning. However, as we make progress in combining our creative mathematics with a respectful etiquette, and the teachers we work with make progress in helping children enjoy a disciplined mathematics, perhaps in a way they never did, our project should be extended to more and more threatening situations, i.e., schools where violence has established itself more substantially. Does disobedience knit people together as Richard Sennet (1980) claims, or does it undermine the group solidarity as most teachers seem to believe? Perhaps the key goal of the research project is to find a way of describing teachers' ways of thinking while teaching that which will be acceptable to them and explanatory to the specialists whose advice has been rejected by teachers.

Perhaps the key to the practical art of being a resource person is that, in the midst of teachers' struggles with the misunderstandings children exhibit of many of their teachers' efforts to explain and demonstrate what is obvious to them, they will discover some of the (not-so-obvious) subtleties (niceties) of numbers that are usually overlooked in their interest of calculation algorithms. For example, understanding Dienes' multi-base blocks is the kind of thing that can't be explained but comes in a flash when you've

done enough work with several systems, not necessarily computat... For example, just playing "super block" with children using two or three bases other than ten can give you a whole new insight into the base ten place-value system, especially borrowing and carrying. It isn't that teachers feel the need for understanding borrowing and carrying ("re-naming") any better. They are all quite competent in adding and subtracting in the decimal system. But when an integrating insight flashes, one tends to ask, "Why didn't anyone tell me about this?" People usually don't know that they lack an integrating understanding until they have it. When you move to a new city, you quickly memorize a half dozen or so specific trips, but it takes much longer to integrate them into one mental map which permits you to navigate quite freely, to explain to people how to get somewhere that you hadn't memorized, and to ask intelligent questions about how to get places. Before this integration occurs, you might know what you're lacking having experienced it in another place. Somehow, arithmetic is thought to be technique - not like any other territory.

The field of teacher education, especially in-service teacher education, is so full of dogmas that it may be necessary for us to disavow most of them and to search our own work carefully to find what our dogmas are. We do not subscribe to the dogma that before teachers can recognize creative mathematics in children and award it appropriate support, they have to experience such creativity themselves. It seems helpful to me when I teach that I have had some minor experiences of creative problem solving. However, that does nothing at all to prove that it is necessary for other teachers, especially if they are turned off by the prospect of working on unfamiliar mathematics problems. Also, I do not accept the prevelant view that teachers need to consider what their objectives are before they set out to change their practices.

That is a most difficult and abstract task. It is [illegible] is commonly recommended, presumably because, when people detect differences in perception, they usually tie them to differences of objectives. To spend time discussing objectives, however, is not to change the way teachers perceive events in the classroom, but the way they perceive events in the discussion.

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